Fast ion transport studies in toroidal plasmas with magnetic islands

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I. Introduction. In burning plasmas it is imperative to have good confinement of fast ions which should provide the required heating. They can arrise due to NBI heating or as byproduct of fusion reactions. The transport of these particles can be described by neoclassical theory in the low collisionality regime which has been developed for plasmas with nested magnetic flux surfaces. But in toroidal plasmas magnetic islands are usually present at low-order rational magnetic surfaces which create not nested magnetic surfaces. This is expected to modify the transport of particles. It is then interesting to know how the islands affect the confinement of fast ions. Although energetic particles are prone to excite MHD modes which in turn alter the background magnetic fields and therefore affect the transport, we here concentrate in just the studying transport given the fields. A similar study has been carried out recently for runaway electrons in which the computations with a guiding center (GC) approximation were compared with full orbit (FO) description [1]. Our approach considers a population of test particles in the presence of a toroidal magnetic field including islands and follow their motion subject also to collisions with a Maxwellian plasma background consisting of electrons and a single species of ions. The collisions are described by stochastic operators [2]. Particle orbits are obtained by numerically solving stochastic equations and the evolution of the particle ensemble when crossing the island region is analyzed.

Particle dynamics is followed using both a guiding center approximation with a code in toroidal geometry [3] and a full orbit description with the code KORC [4]. In this way we compare the predictions of both approaches in order to determine how important FO effects are for fast ions, given that GC simulations are much faster to perform. The presence of an electric field is included associated with the island, as well as the rotation of the islands is also included in the study.

II. Magnetic field geometry and islands The magnetic field model with nested circular toroidal flux surfaces is given, in toroidal coordinates, by $\mathbf{B}(\mathbf{r},\theta) = \left[\mathbf{B}_\zeta(\mathbf{r},\theta)\hat{\mathbf{e}}_\zeta + \mathbf{B}_\theta(\mathbf{r},\theta)\hat{\mathbf{e}}_\theta\right]$ where

$$B_\zeta(r,\theta) = Bo/(1+\varepsilon\cos\theta), \quad B_\theta(r) = \kappa\varepsilon B_0/[q(r)(1+\varepsilon\cos\theta)]. \tag{1}$$

with $\varepsilon = r/R_0$, $\kappa = \hat{\mathbf{B}}_\zeta \cdot \hat{\mathbf{J}}_\zeta$ and $q(r) = q_0\left(1+\frac{r^2}{\varepsilon^2}\right)$. The island filed given by $\delta B = \nabla\times\delta A$ where $\delta A = \tilde{\alpha}(r,\theta,\zeta)R_0\mathbf{B}(\mathbf{r},\theta)$, $\tilde{\alpha}(r,\theta,\zeta) = \alpha(r)\cos\varphi_{mn}$, with $\varphi_{mn} = n\zeta - \kappa m\theta + \omega_{m,n}t$, $\alpha(r) = \alpha^*(r/r^*)[(a-r)/(a-r^*)]^p$. The corresponding island width is $\delta r_{mn} \approx \sqrt{\frac{4q(r)b(r)R_0}{nq'(r)}}$

where $b(r) = m\alpha(r)R_0/r$

In the GC code it is only required the magnitude $B(r,\theta)$ which is taken as $B(r,\theta)/B_0 = 1-\varepsilon\cos\theta$.

Fig. 1 shows Poincaré sections of the magnetic field lines for two perturbation amplitudes $\delta B/B_0 = 10^{-4}$ and $\delta B/B_0 = 10^{-3}$ corresponding to island sizes $\delta r = 0.01m, \delta r = 0.03m$, for the mode $(m,n) = (2,1)$ when it is located at $r^*/a = 0.5$

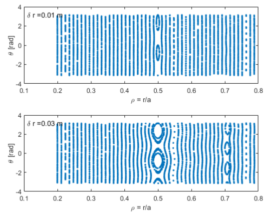

Figure 1: 2/1 islands for $\delta = 0.01$ and 0.03.

III. Particle transport simulations Single particle trajectories are followed using both FO code (KORC [2]) and GC code (gcaf) which include random collisions up to times of order of slowing down time.

A DIII-D-like tokamak is used with $B_0 = 2$ T, $R_0 = 1.5$ m, $a = 0.5$ m, $q_0 = 1$, $q_a = 5(\varepsilon = a/2)$, and plasma current $I_p \sim 300$ kA, while the current direction gives $\kappa = -1$. The background electron and ion temperatures are held fixed at $T_e = 1\,keV, T_i = 200\,eV$.

An ensemble of monoenergetic and mono-pitch-angle particles starting on a single flux surface inside the island region is followed and the flux of particles crossing a surface at a radius $r = 0.54a > r^*$ (external to the island region) is computed.

In all simulations a $m/n = 2/1$ island is assumed in the rational surface where $q(r^*/a = 0.5) = 2$. The initial surface is at $r_{in}/a = 0.38 < r^*/a$ (inside the island region). Simulations cover the parameter ranges (pitch angle $\eta = 0$): $n = 10^{19} - 10^{20} m^{-3}$, $\mathscr{E} = 10$ keV -50 keV.

In order to check compatibility of FO and GC codes we compared the thermalization times for both. As shown in Fig. 2, the collisional thermalization times are consistent ($t \approx 13\,ms$) in FO (a) and GC (b) simulations, for $n = 10^{20} m^{-3}, \mathscr{E} = 10$ keV.

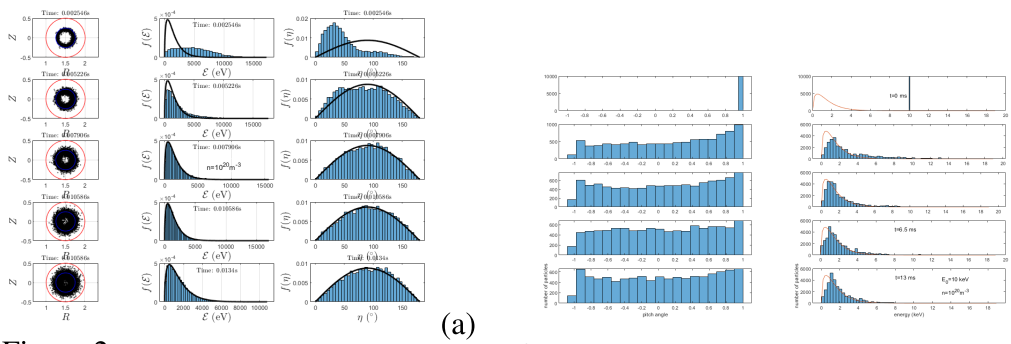

Figure 2: Distributions tend to Maxwellian in $\mathscr{E}$ and constant in pitch angle in same time for FO and GC codes.

Knowing this, we first show for the FO case, in Fig. 3 the effect of the island on the number of crossing particles (N_{out}) for two island sizes and three densities. It is clear that the particle flow increases as the island grows but the effect is less evident for higher densities. However, for the higher energy this trend tends to reverse. On the other hand, the results for the GC code can be seen in Fig. 4, when N_{out} is computed for two different island sizes for $n = 10^{20} m^{-3}$ and two ion energies. These results show a larger flux for the wider island in agreement with Fig. 3.

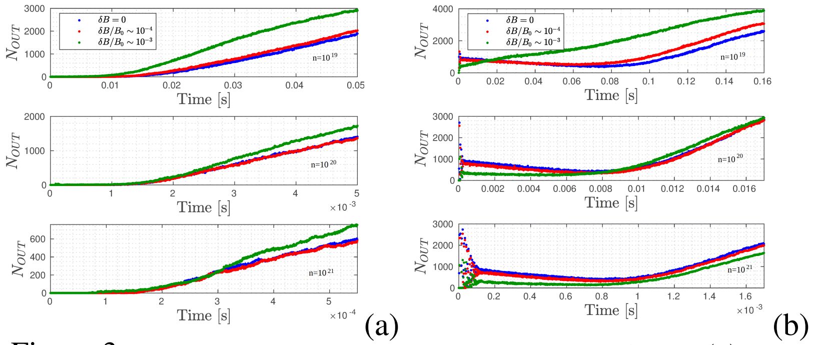

Figure 3: FO flows for 3 densities & 2 island sizes: (a) $\mathscr{E} = 10, (b) 50$ keV.

From N_{out} one can obtain a particle flux by fitting a quadratic to $N_{out}(t)$ and defining the flux as $\Gamma = dN_{out}/dt$. This can show better the flux changes.

Now we include the effect of island rotation $\omega \neq 0$. An important difference between FO and GC results is that, for the GC there is practically no difference when the rotation is in the co- and counter- direction with $J(\omega{<}0, {>}0)$, whereas the FO fluxes show a significant difference as shown in Fig. 5, for wide and narrow islands.

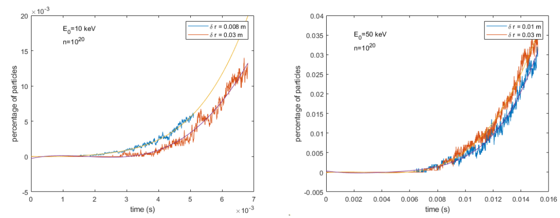

Figure 4: GC flows for mid-density & 2 island sizes ($\mathscr{E} = 10, 50$ keV).

Concentrating then in the FO results, we varied the rotation frequency from 100 Hz to 100 kHz obtaining that there is first an increase in particle flux and then a decrease with a maximum at $\omega \sim 10^4$ (see Fig. 6 for $\delta = 0.03$m; similar behavior is found for $\delta = 0.01$ m). This may point to a resonant process.

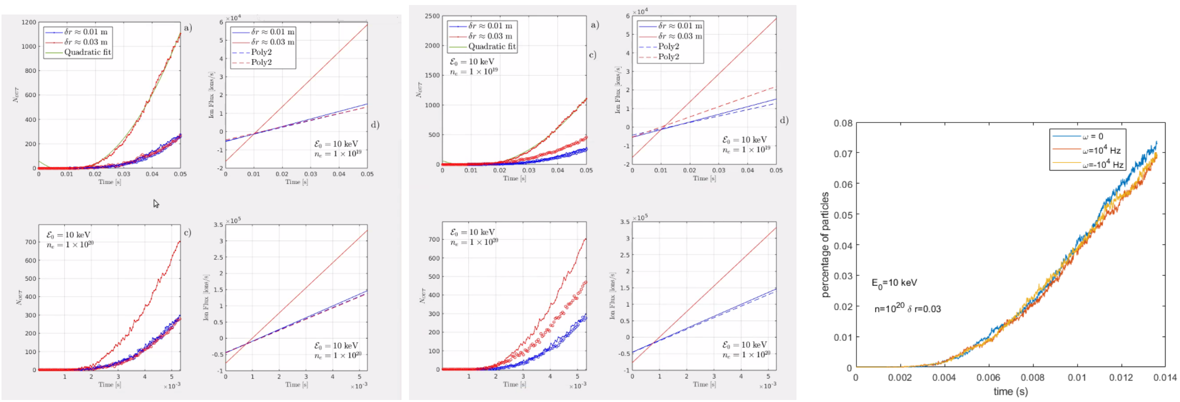

Figure 5: Island rotation for FO (left $\omega > 0$, middle $\omega < 0$) and GC (right). No rotation is dashed.

IV. Electric fields of island. The presence of the island introduces electric potentials in and around the island. The potential can be modeled by

$$\phi(r, \theta, \zeta) = -\phi_0 \left[\frac{1}{1+\exp\{-k(r-r_o-d(1-\cos^2\varphi))\}} - \frac{1}{1+\exp\{k(r-r_o+d(1-\cos^2\varphi))\}} - 1 \right]$$

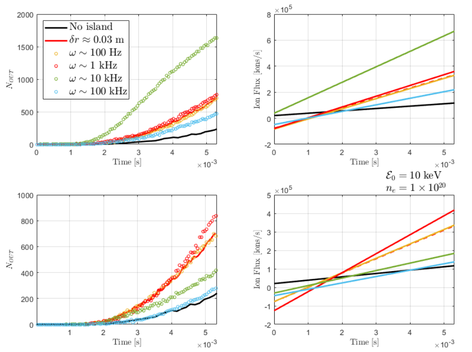

Figure 6: Ion fluxes for various rotations and wide island.

where, $\varphi = \frac{m}{2}\kappa\theta - \frac{n}{2}\zeta + \omega_{m,n} - \omega t$, and d being the half-width of the region where the electric potential is maximum and has a plateau. In all our simulations the latter corresponds to the half-width of the magnetic island, $d = \delta r_{mn}$. The electric field $\mathbf{E} = -\nabla\phi$, has three components; the parameters d and k can change its form. When the electric field is included but with no island rotation the fluxes are increased as sen in Fig. 7. Continuous lines are for no electric field, red is for wide island while blue is for narrow island. The effect of a 3-dimensional electric field is to make ions cross the island region more efficiently, contrary to a purely radial electric field which tends to reduce radial flux due to poloidal $E \times B$ drift.

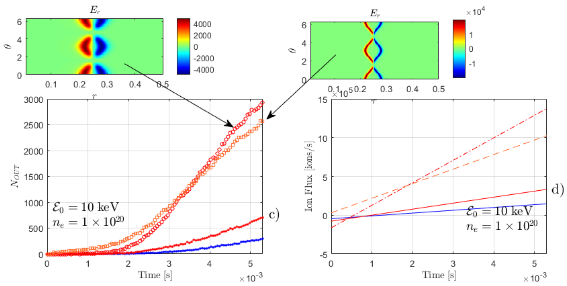

Figure 7: Electric field for two gradients increases flux.

V. Conclusions. The presence of magnetic islands modifies the radial flux of fast ions across the rational surface containing the island, in a variety of ways: (a) Ions following the magnetic field lines are favored to cross the islands, so wider islands produce larger fluxes as compared to flux without island. This is true for FO and GC simulations. (b) More collisional plasmas (higher n) have reduced increase of the radial flux since collisions hinder particles to follow island lines to outer flux surfaces. Particles of higher energy are less sensible to this effects since their collisionality is lower. (c) A rotating island can produce increment ($\omega < 0$) or reduction ($\omega > 0$) of flux; this is a FO orbit effect since GC simulations do not show modification. (d) As function of ω flux first increases and then decreases indicating a resonant process with the rotation at $\omega \sim 10^4 Hz$. (e) The electric field associated with the island produces also an increase of the radial flux. This is a purely 3D effect of the E-field since a radial E-field reduces radial fluxes.

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