Reconnection Heating in the Solar Corona with Compressional Magnetic Fluctuations

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Previous studies have shown that reconnection turbulence is a promising candidate for explaining the solar corona heating rate. Commonly, plasma with reduced mass ratio and β value are used for faster convergence. While reconnection processes rely primarily on shear magnetic fluctuations, at typical β values, compressional magnetic fluctuations can affect growth rates and heating rates. Similiarly, even though compressional magnetic fluctuations tend not to have a large effect in core fusion plasmas, they can affect electromagnetic modes in the tokamak pedestal and the LAPD high-β experiments. [1]

We first use the local version, which uses the compressional magnetic fluctuations, of the gyrokinetic code GENE with realistic β value and Hydrogen mass ratio to verify the heating rate of the reconnection turbulence matches the observed solar corona heating rate, and confirms extrapolations made in earlier studies. [2]

For studying reconnection heating more comprehensively, the radially global version of GENE needs to be used. [3] This alleviates the periodic constraint on the radial direction, and enables the use of Dirichlet or Neumann boundary conditions in the radial direction. To this end, the radially global gyrokinetic framework including compressional fluctuations is derived and implemented in the GENE code.

Due to the usage of finite-element radial base function, the magnetic potential in the two directions perpendicular to the background magnetic field needs to be computed separately. This decouples the $B_\parallel$ from its gyroaveraged quantity $\bar{B}_\parallel$, thus a new gyroaverage procedure for the compressional magnetic field is also implemented.

From the Poisson's equation and Ampere's law, we have: [4]

$$\nabla^2\Phi_1 = 4\pi\rho \qquad \nabla\times\vec{B}_1 = \frac{4\pi}{c}\vec{j}$$

$$\rho(\vec{x}) = \sum_\sigma q_\sigma \int F^*_\sigma(\vec{x},\vec{v})d^3v \qquad \vec{j}(\vec{x}) = \sum_\sigma q_\sigma \int \vec{v}F^*_\sigma(\vec{x},\vec{v})d^3v$$

$$F^*_\sigma(\vec{x},\vec{v}) = T^*F_\sigma(\vec{X},v_\parallel,\mu,\theta)$$

$$T^*f_{1\sigma} = f_{1\sigma} + \frac{1}{B_0}\left[\frac{\mathbf{B}^*_0}{B^*_{0\parallel}}\left(\Omega_\sigma\frac{\partial F_{0\sigma}}{\partial v_\parallel} - \frac{q_\sigma v_\parallel}{c}\frac{\partial F_{0\sigma}}{\partial\mu}\right)\cdot\tilde{\mathbf{A}}_1 + \left(q_\sigma\tilde{\Phi}_1 - \mu\bar{B}_{1\parallel}\right)\frac{\partial F_{0\sigma}}{\partial\mu}\right]$$

Where the T^* is the pullback operator that transform between the real coordinate and the gyrocenter coordinate. The gyroaverage on the compressional magnetic field fluctuation $B_{1\parallel}$ is treated as:

$$\mu\langle B_{1\parallel}\rangle = \frac{q}{c}v_\perp\langle \mathbf{A}_1\cdot\mathbf{c}\rangle, \qquad \mathbf{c} = -\sin\theta\hat{\mathbf{e}}_1 + \cos\theta\hat{\mathbf{e}}_2$$

From these We obtain the normalized global field equations:

$$\hat{\Phi}_1(\mathbf{x}) = \hat{P}^{-1}\sum_\sigma \hat{n}_{0\sigma}(x_0)\hat{q}_\sigma\left[\pi\iint \hat{B}^*_{0\parallel}(\mathscr{G}^\dagger\hat{F}_{1\sigma})d\hat{v}_\parallel d\hat{\mu} + \frac{\hat{n}_{p\sigma}}{T^2_{p\sigma}}\hat{B}_0\int\hat{\mu}(\mathscr{G}^\dagger\mathscr{G}\hat{B}_{1\parallel})e^{\frac{-\hat{\mu}\hat{B}_0}{T_{p\sigma}}}d\hat{\mu}\right]$$

$$\hat{A}_{1\parallel}(\mathbf{x}) = \hat{A}^{-1}\sum_\sigma\left[\frac{\beta_{\mathrm{ref}}}{2}\hat{q}_\sigma\hat{n}_{0\sigma}(x_0)\hat{v}_{T\sigma}(x_0)\pi\iint\hat{B}^*_{0\parallel}(\mathscr{G}^\dagger\hat{g}_{1\sigma})\hat{v}_\parallel d\hat{v}_\parallel d\hat{\mu}\right.$$

$$(\hat{\mathbf{e}}_1\hat{\partial}_{\bar{y}} - \hat{\mathbf{e}}_2\hat{\partial}_{\bar{x}})\hat{B}_{1\parallel} + \frac{\beta_{\mathrm{ref}}}{2}\hat{B}_0^{\frac{3}{2}}\sum_\sigma\hat{q}_\sigma\hat{n}_{0\sigma}\hat{v}_{T\sigma}\pi\iint(-\hat{\mathbf{e}}_1\mathscr{G}^{s\dagger}\mathscr{G} + \hat{\mathbf{e}}_2\mathscr{G}^{c\dagger}\mathscr{G})\hat{B}_{1\parallel}\frac{\hat{F}_{0\sigma}}{T_{p\sigma}}\hat{\mu}^{\frac{3}{2}}d\hat{v}_\parallel d\hat{\mu}$$

$$= -\frac{\beta_{\mathrm{ref}}}{2}\hat{B}_0^{\frac{3}{2}}\sum_\sigma\hat{q}_\sigma\hat{n}_{0\sigma}\hat{v}_{T\sigma}\pi\left[\frac{\hat{B}^*_{0\parallel}}{\hat{B}_0}\iint(-\hat{\mathbf{e}}_1\mathscr{G}^{s\dagger} + \hat{\mathbf{e}}_2\mathscr{G}^{c\dagger})\hat{F}_{1\sigma}\sqrt{\hat{\mu}}d\hat{v}_\parallel d\hat{\mu}\right.$$

$$\left. + \frac{\hat{q}_\sigma}{\hat{T}_{0\sigma}}\iint(-\hat{\mathbf{e}}_1\mathscr{G}^{s\dagger}\mathscr{G} + \hat{\mathbf{e}}_2\mathscr{G}^{c\dagger}\mathscr{G})\hat{\Phi}_1\frac{\hat{F}_{0\sigma}}{T_{p\sigma}}\sqrt{\hat{\mu}}d\hat{v}_\parallel d\hat{\mu}\right]$$

Where the coupled $\Phi - B_\parallel$ equation is implemented into GENE as:

$$\begin{aligned}[C_1]\hat{\Phi}_1 + [C_2]\hat{B}_{1\parallel} &= RHS_1 \\ [C_3]\hat{\Phi}_1 + [I]\hat{B}_{1\parallel} &= RHS_2\end{aligned} \tag{1}$$

To implement the coupled Φ-$B_\parallel$ solver, two new gyromatrices $\mathscr{G}^s_{in}$ and $\mathscr{G}^c_{in}$ are introduced along with the standard gyromatrix $\mathscr{G}$:

$$\mathscr{G}_{in}(x,k_y,z,\mu) = \frac{1}{2\pi}\int_0^{2\pi}\Lambda_n(x_{(i)} - r^1)e^{-ik_yr^2}d\theta$$
$$\mathscr{G}^s_{in}(x,k_y,z,\mu) = \frac{1}{2\pi}\int_0^{2\pi}\sin\theta\Lambda_n(x_{(i)} - r^1)e^{-ik_yr^2}d\theta$$
$$\mathscr{G}^c_{in}(x,k_y,z,\mu) = \frac{1}{2\pi}\int_0^{2\pi}\cos\theta\Lambda_n(x_{(i)} - r^1)e^{-ik_yr^2}d\theta$$

These two new gyromatrices tends to have large condition numbers for some set of parameters, and debugging is ongoing to improve the reliability of this implementation. Once this task is complete, it will be deployed for the calculation of the kinetic ballooning modes in the pedestal, where large gradients are present and the compressional magnetic fluctuations could affect their growth rates.

For benchmarking with the local version of GENE, using the local limits:

$$\Lambda_n(x) \to e^{ik_xx}, \qquad \Lambda_n(X) \to e^{ik_xX}, \qquad \sum_n \to \sum_{k_x}$$

We can obtain:

$$\mathscr{G}^c g_{1\sigma} = \left\langle \cos\theta g_{1\sigma}(\mathbf{x}-\mathbf{r}) \right\rangle = \sum_{k_x,k_y} e^{ik_x x} e^{ik_y y} g_{1\sigma}(k_x,k_y,z)(-i) M_c J_1(k_\perp \rho)$$

$$\mathscr{G}^s g_{1\sigma} = \left\langle \sin\theta g_{1\sigma}(\mathbf{x}-\mathbf{r}) \right\rangle = \sum_{k_x,k_y} e^{ik_x x} e^{ik_y y} g_{1\sigma}(k_x,k_y,z)(-i) M_s J_1(k_\perp \rho)$$

$$M_c = \cos\phi = \frac{1}{k_\perp}\big(k_x\sqrt{g^{11}} + k_y \frac{g^{12}}{\sqrt{g^{11}}}\big) \quad M_s = \sin\phi = \frac{1}{k_\perp}\big(k_y\sqrt{\frac{\gamma_1}{g^{11}}}\big)$$

$$\mathscr{G} \to J_0(k_\perp \rho) \qquad \mathscr{G}^c \to \pm i M_c J_1(k_\perp \rho) \qquad \mathscr{G}^s \to \pm i M_s J_1(k_\perp \rho)$$

$+$ for $\mathscr{G}'_{c/s}$ on fields, $-$ for $\mathscr{G}_{c/s}$ on distribution functions.

References

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