Nonlinear evolution of the parametric instability in the high latitude solar wind

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Since the first observations (see, e.g., [1]), it has become clear that the solar wind is a highly turbulent medium. In fast streams, the major component of the turbulent fluctuations is dominated by large amplitude, arc-polarized (i.e., with a constant magnetic field intensity), fluctuations propagating outward with respect to the Sun, exhibiting a high correlation between velocity and magnetic field fluctuations and low density and magnetic field intensity perturbations.

Strictly speaking, such fluctuations are not Alfvén waves, that also have very similar characteristics, but small amplitude and a different polarization. However, an arc-polarized fluctuation with Alfvénic correlation is an exact solution of the ideal MagnetoHydroDynamics (MHD) equations, even in a nonlinear, compressible, case. As a consequence, such fluctuations should propagate undistorted in the solar wind during its expansion in the heliosphere.

On the contrary, the Alfvénic correlation of the turbulent fluctuations decreases with the distance from the Sun. Bavassano and Bruno [2], Grappin et al. [3], showed that the normalized cross-helicity, which measures the degree of Alfvénicity of the fluctuations, decreases while the solar wind moves inside the heliosphere, thus indicating that inward propagating Alfvénic fluctuations, which tend to destroy the Alfvénic correlation, are continuously produced during the expansion of the wind. Such production is known to be possible because of the magnetic field inhomogeneities, but it is much less probable in the case of fast solar wind streams, where the magnetic field is more homogeneous. At the same time, also an increasing level of compressive and magnetic field intensity fluctuations is observed with increasing distance from the Sun. This indicates that, in spite of the fact that arc-polarized, Alfvénic fluctuations are an exact solution of compressible MHD equations, they have tendency to decay due to some mechanism and produce counter-propagating Alfvénic fluctuations, along with compressive perturbations.

A possible mechanism that can explain such phenomenology is the parametric instability, that was studied in the past in a variety of situations (see, e.g., [4], [5], [6], [7]). In the present work, we built up an initial condition which should be more similar to the one found in the solar wind, by exciting an anisotropic spectrum of fluctuations with Alfvénic correlation, globally having a magnetic field intensity constant everywhere in the computational domain at $t = 0$.

The initial condition is: $\rho(x,y) = \rho_0$; $\mathbf{B}(x,y) = B_0\hat{e}_x + \delta\mathbf{B}(x,y)$; $\mathbf{v}(x,y) = -\delta\mathbf{B}(x,y)/\sqrt{\rho}$

where $\delta\mathbf{B}$ are turbulent fluctuations of the magnetic field, defined as:

$$\delta B_x(x,y) = \frac{\partial A(x,y)}{\partial y} - < \frac{\partial A(x,y)}{\partial y} >$$
$$\delta B_y(x,y) = -\frac{\partial A(x,y)}{\partial x}$$
$$\delta B_z(x,y) = \sqrt{B_T^2 - [B_0 + \delta B_x]^2 - \delta B_y^2}$$

where B_T is the total magnetic field intensity (chosen as a parameter) and the z-component $A(x,y)$ of the potential vector is defined in the spectral space as:

$$\hat{A}(k_x,k_y) = \begin{cases} A_0 \frac{\exp[-6(k_y/k_c)^6 + i\phi(k_x,k_y)]}{\sqrt{1+(k_y/k_{Y0})^4}} & for|k| < k_{x,max} \\ 0 & for|k| > k_{x,max} \end{cases}$$

$k_{x,max} = k_{0y} = 4$ and $k_c = 50$ are parameters chosen in such a way that the resulting spectrum of A and $\mathbf{B}$ are anisotropic, to mimic the field observed in the solar wind. Such a solution represents a turbulent Alfvénic fluctuation propagating in one direction with respect to the background magnetic field (a $\mathbf{Z}^-$ Elsässer mode) in which the total (background plus fluctuating fields) magnetic field intensity: $|\mathbf{B}|^2 =$ constant at $t = 0$.

Such an initial condition was used to initialize a numerical, pseudo-spectral, fully-periodic, parallel code which solves the compressible, dimensionless, polytropic MHD equations:

$$\frac{d\rho}{dt} + \rho(\nabla\cdot\mathbf{v}) = 0$$
$$\frac{d\mathbf{v}}{dt} = -\beta\rho^{\gamma-2}\nabla\rho + \frac{1}{\rho}(\nabla\times\mathbf{B})\times\mathbf{B} + \frac{\nu}{\rho}\left\{\nabla^{2n}\mathbf{v} + \frac{1}{3}\left[\nabla^{2(n-1)}\nabla(\nabla\cdot\mathbf{v})\right]\right\}$$
$$\frac{\partial\mathbf{B}}{\partial t} = \nabla\times(\mathbf{v}\times\mathbf{B}) + \eta\nabla^{2n}\mathbf{B}$$

in a $2+1/2$-D domain: $\{(x,y)\} \in [0,2\pi L]\times[0,2\pi L]$. In the above equations, ρ represents the density, $\mathbf{v}$ the velocity field (in units of the Alfvén speed), $\mathbf{B}$ the magnetic field, $\beta = c_s^2/c_A^2$ the plasma beta (ratio between the sound speed and the Alfvén speed), γ the polytropic index, and n the hyperviscosity order. The lengths are normalized to the box dimension L and the times to L/c_A. We ran our simulations for two values of $\beta = 0.5$ and 1.5, which are typical values observed for the solar wind, $\gamma = 5/3$ and $n = 2$ (fourth order hyperviscosity-resistivity).

Figures 1 show the quantities: $r(t) = \sqrt{\left\langle\left(\frac{\delta\rho}{\rho_0}\right)^2\right\rangle}$, $m(t) = \sqrt{\left\langle\left(\frac{\delta|\mathbf{B}|}{<|\mathbf{B}|>}\right)^2\right\rangle}$ and $e^{\pm}(t) = \frac{1}{2} < (\delta\mathbf{Z}^{\pm})^2 >$, which represent rms values of the density, magnetic field intensity and Elsässer variables $\mathbf{Z}^{\pm}$ energies, respectively, integrated over the whole simulation domain as a function of time t, for the case $\beta = 0.5$ (left panel) and $\beta = 1.5$ (right panel). It is visible as the instability

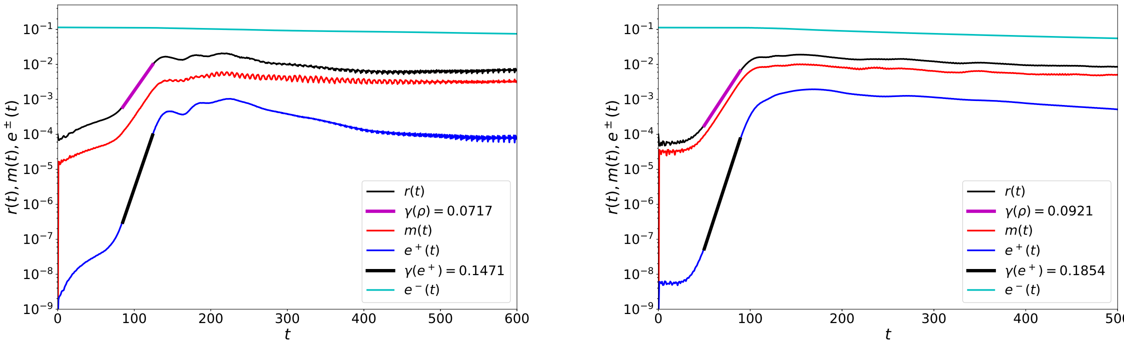

Figure 1: *Time evolution of the integrated quantities* r*,* m*,* $e^{\pm}$ *for* $\beta = 0.5$ *(left panel) and* 1.5 *(right panel)*

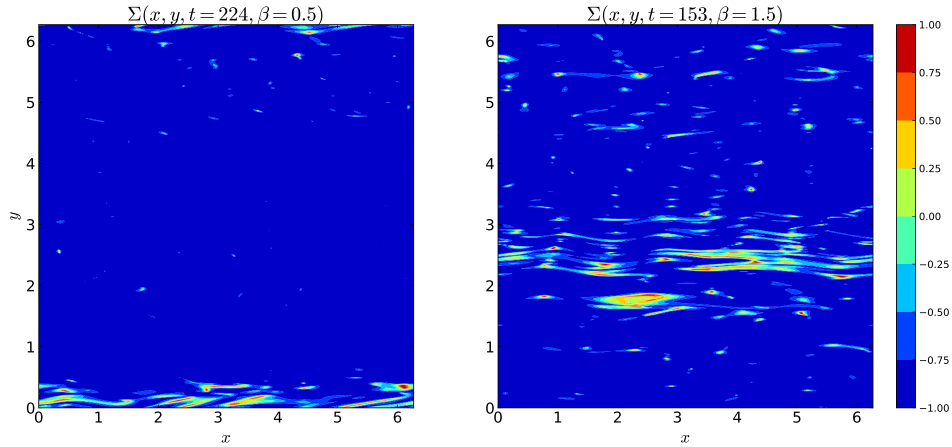

Figure 2: *Snapshot at* $t = 224$ *(left panel) for the case* $\beta = 0.5$ *and* $t = 153$ *(right panel) for the case* $\beta = 1.5$*, of the "localized cross-helicity"* Σ*, defined in the text.*

triggers an exponential increase of r, m and e^+. The latter represents a backscattered Elsässer mode. The growth rates γ for the three quantities are shown in the plots, as well. The growth rate for the case $\beta = 1.5$ is slightly larger than the one for the case $\beta = 0.5$, as opposite to what happens in the monochromatic case [4].

Figures 2 show the "localized" cross-helicity, i.e. the quantity: $\Sigma(x,y,t) = \dfrac{|\delta \mathbf{Z}^+|^2 - |\delta \mathbf{Z}^-|^2}{|\delta \mathbf{Z}^+|^2 + |\delta \mathbf{Z}^-|^2}$ at times corresponding approximately at the maximum growth of the instability. The two panels show the plots for the two different β values. The quantity Σ assumes values between $+1$, corresponding to maximum $\mathbf{Z}^+$ and vanishing $\mathbf{Z}^-$, and -1, corresponding to maximum values of $\mathbf{Z}^-$ and vanishing values of $\mathbf{Z}^+$. The instability brings to the formation of backscattered fluctuations $\mathbf{Z}^+$ only in strongly localized positions in the domain, often organized in "strips", which propagate at subsequent times mainly along the mean magnetic field direction $B_0 \hat{e}_x$.

Finally, the plots in Fig. 3 show some cuts in the x and y directions, for the density and magnetic field intensity fluctuations, for the $\beta = 0.5$ case (upper panels) and $\beta = 1.5$ case (lower panels). For both values of β, it is visible that the fluctuations are in a rather turbulent state

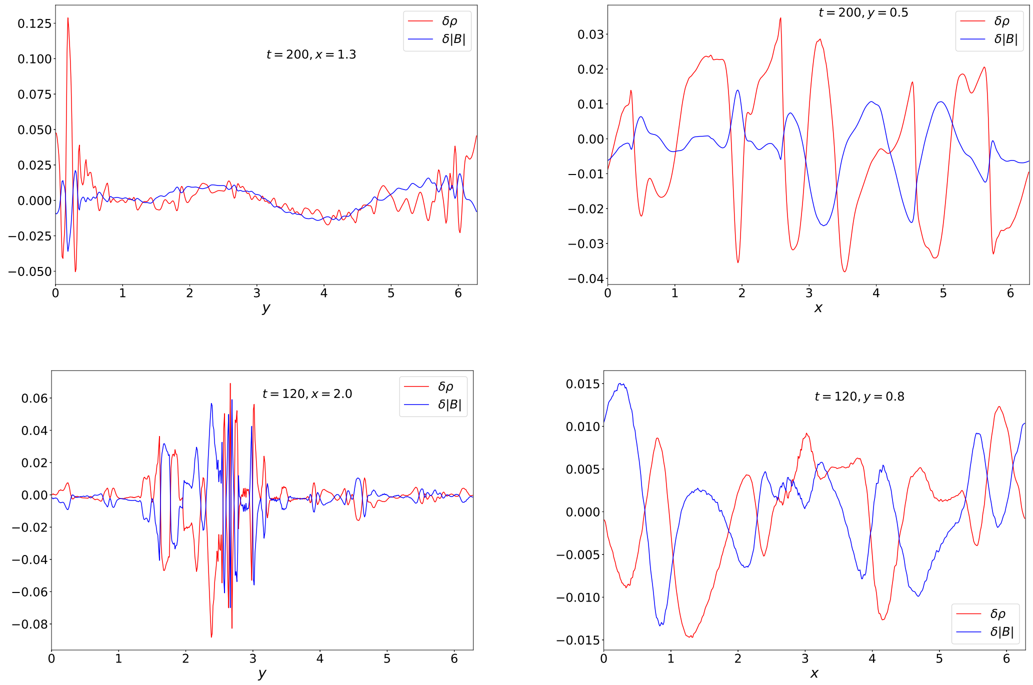

Figure 3: *Cuts of the density and magnetic field intensity profiles along the x and y directions at the points of the numerical domain indicated in the legends at given times, for the case* $\beta = 0.5$ *(upper panels) and* $\beta = 1.5$ *(lower panels).*

dominated by a marked anti-correlation between density and magnetic field intensity fluctuations, which indicates the presence of nearly pressure-balanced structures. Such structures have been detected everywhere in the solar wind (e.g., [2], [8]).

In summary, our numerical simulations are able to recover several features already observed in the solar wind: 1) the parametric instability confirms itself as a suitable mechanism to produce the observed generation of inward-propagating Alfvénic fluctuations and compressive fluctuations; 2) pressure-balanced structures are obtained in the simulations as a by-product of the parametric instability; 3) we observe the formation of coherent structures which persist during the propagation of the fluctuations.

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