On the nonlinear excitation of electron plasma waves at the plasma edge in the O1-mode ECRH experiments

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1. Introduction. The O1-mode ECRH technique is considered for the local electron heating providing the neoclassical tearing mode control in ITER. Until very recently the propagation and absorption of microwaves were believed to be predictable in detail. However, as it was observed in a number of the O1-mode ECRH experiments [1,2] the pump ordinary wave can suffer from anomalous scattering, which was explained by the nonlinear excitation of trapped upper hybrid waves at the local maximum of a non-monotonic density profile [3]. As shown in the present paper, ordinary microwaves can also suffer from nonlinear parametric phenomena at the plasma edge, where a transport barrier is usually observed. The presence of a large density gradient have a significant impact on the properties of waves in the low hybrid (LH) frequency range leading to new transparency windows that are absent in the homogeneous plasma [4]. These new modes can be 2D localized along the direction of a plasma inhomogeneity due to gradient effects and along the magnetic field due to magnetic ripples. The instability power threshold leading to the 2D localized wave excitation is much less than MW and can be overcome in future O1-mode ECRH experiments at ITER.

2. Intermediate frequency wave trapping in strongly inhomogeneous plasmas. The usual approach to the analysis of intermediate frequency waves in inhomogeneous magnetized plasmas is the WKB approximation, which leads to the same conclusions on the wave transparency regions as the homogeneous plasma theory. However, strong plasma inhomogeneity at the plasma edge combined with a large value of the non-diagonal dielectric tensor component can lead to a significant change in the wave transparency [4,5], creating new transparency regions. The wavelength in this case remains much smaller than the plasma inhomogeneity scale length and therefore the effect can be accounted for in the WKB approximation modified by adding terms proportional to the derivatives of the dielectric tensor components [4]. To illustrate this phenomenon in the tokamak edge transport barrier (ETB), we introduce the local Cartesian coordinate system (x, y, z) with x being related to the flux surface label, y and z being the coordinates perpendicular to and align with the magnetic field. The magnetic field in a narrow layer in the ETB has the form

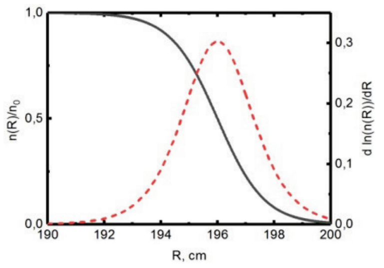

Fig.1. Density profile normalized to the density at the magnetic axis (solid line), and the profile of its derivation (dashed line).

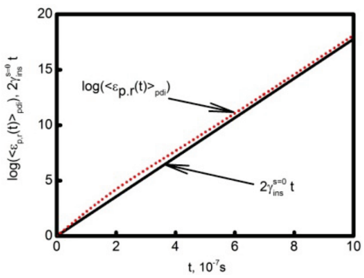

Fig. 2. The amplification coefficients obtained numerically (dotted curve) and predicted analytically (solid curve). $s=0$, $P_0 = 1\mathrm{MW}$.

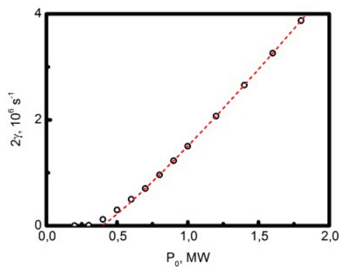

Fig. 3. Dependence of the growth rate on the pump power, $w = 2$ cm. The solid curve is Eq. (6). The scattered circles are numerical solution $\delta \simeq 0.3\%$

$B = \bar{B}\left(1-\delta(x,y)\cos(Nz/R)\right)$ with N being the number of toroidal coils, R - the major radius of tokamak and δ is the amplitude of magnetic ripples. The amplitude of the electrostatic electron plasma (EP) wave $\phi(\mathbf{r}) = \psi(x,z)\exp\left(iq_y y + i\omega_L t\right)/2 + c.c.$ is described by Poisson's equation $\hat{D}_{EPW}\psi = \left(\varepsilon(\omega_L)\Delta_\perp + \partial_x \varepsilon(\omega_L)\partial_x + \partial_x g(\omega_L) q_y + \eta(\omega_L)\partial_{zz}\right)\psi = 0$ where ε, g, η are the components of the cold-plasma dielectric tensor, $\Delta_\perp = \partial_{xx} - q_y^2$, $\partial_\zeta = \partial/\partial\zeta$, $\zeta = x, y$ and the term $\sim \partial_x \varepsilon(\omega_L)$ being much smaller than the term $\sim \partial_x g(\omega_L)$ by a factor of $\omega_L/\omega_{ce} << 1$ will be further neglected. The term $Q = \partial_x g/\varepsilon|_{\omega_L}$ in the operator $\hat{D}_{EPW}$ depends on the coordinates x and z. Taking the density profile close to that expected in ITER in the ETB [6] we plot it in figure 1 with its spatial derivative, which has a local maximum at x_m. Since the function Q depends on the magnetic field, it has also a local minimum along the toroidal direction at $z = 0$ between the two adjacent toroidal magnetic field coils where the pump power is launched. Then, we approximate the function Q by quadratic dependencies over both coordinates $Q \approx Q_0(1-(x-x_m)^2/(2l_x^2) + z^2/(2l_z^2))$ around $x = x_m$, $z = 0$. Using this expansion yields

$$\hat{D}_{EPW}\psi \simeq \varepsilon(\omega_L, x_m)\left(\partial_{xx} + Q_0 q_y - q_y^2 - K_x^4 (x-x_m)^2\right)\psi - \left|\eta(\omega_L, x_m)\right|\left(\partial_{zz} - K_z^4 z^2\right)\psi = 0 \qquad (1)$$

where $K_{x,z} = const$. The solution to equation (1), representing the EPW trapped both along the magnetic field and in the radial direction $\psi(\mathbf{r}) = \psi_{p,r} f_p\left(K_x(x-x_m)\right) f_r\left(K_z z\right)$, $\psi_{p,r} = const$, is expressed in terms of the Hermite polynomials f_p. Substituting $\psi(\mathbf{r})$ in (1) gives the quantization condition for its eigenfrequency

$$D_{EPW}\left(\omega_L^{p,r}\right) = \varepsilon\left(Q_0 q_y - q_y^2 - (2p+1)K_x^2\right) + (2r+1)|\eta| K_z^2 = 0. \qquad (2)$$

These trapped EP waves, which propagate almost across the magnetic field, exist only in strongly inhomogeneous plasmas, where there are regions of transparency for those with a positive poloidal number. If the density gradient is small or the parameter q_y^2 is too large, the plasma for such EPW turns out to be evanescent. It should be noted that the trapped EPW has another noteworthy property. According to (2) its group velocity in the y direction determined as $v_{gy} = \partial D_{EPW} \big/ \partial q_y \big/ \partial D_{EPW} \big/ \partial \omega_L^{p,r} \big|$ takes the zero value at $q_y^* = Q_0 / 2$.

3. Low-threshold parametric excitation of the EPW trapped in the ETB. Given the geometry of future experiments in ITER, we consider an ordinary pump wave propagating perpendicular to the magnetic field along x to the plasma core with its polarization vector being directed mostly along the magnetic field. By means of the WKB approximation it reads $\mathbf{E}_0 = \mathbf{e}_z \sqrt{2P_0/(cw^2)}\; n_x(\omega_0, x)^{-1/2} \exp\left(-(y^2+z^2)/(2w^2) + i\int_0^x k_x(\omega_0, x')dx' - i\omega_0 t\right) + c.c.$ where P_0 - the pump power, w - the width of a beam, *c.c.* - the term derived from the first one by complex conjugation, $k_x(\omega_0) = \omega_0 \eta(\omega_0)/c = \omega_0/c\sqrt{1-\omega_{pe}^2/\omega_0^2}$ - the wave number. Then, we analyze the pump wave decay into the trapped EPW and the side-scattered ordinary wave $\mathbf{E}_s(\mathbf{r}) = \mathbf{e}_z A_s(x)\exp(iq_y y + i\omega_s t)/2 + c.c.$ in the ETB. The daughter waves are described by

$$\begin{cases} \hat{D}_s A_s = \Delta_\perp A_s + \omega_s^2/c^2\eta(\omega_s)A_s = -i\kappa_{nl}\omega_s/c\Delta_\perp E_0^*\psi \\ \hat{D}_{EPW}\psi = i\kappa_{nl}c/\omega_s\Delta_\perp E_0 A_s \end{cases} \tag{3}$$

where $\kappa_{nl} = \omega_{pe}^2/(\omega_0\omega_{ce}\bar{B})$ - the nonlinear coupling coefficient. Solving the first equation in (3) by means of the WKB technique and substituting A_s into the RHS of second equation, we get the equation describing the nonlinear excitation of 2D trapped EPW

$$\hat{D}_{EPW}\psi = \kappa_{nl}^2\Delta_\perp\left(E_0 G_s\left\{\Delta_\perp(E_0^*\psi)\right\}\right) \tag{4}$$

Using the procedure of perturbation theory [7], we arrive at

$$\left(\partial/\partial t + i\Lambda_y\partial^2/\partial y^2\right)\psi_{p,r} = \gamma_0\exp\left(-y^2/w^2\right)\psi_{p,r} \tag{5}$$

where γ_0 - the pumping rate, $\Lambda_y = \varepsilon(\omega_L, x_m)/\partial_{\omega_L}D_{EPW}$ - the EPW diffraction coefficient and <…> - the averaging over the region of the EPW localization. Equation (5) describes the exponential growth of the EPW, which occurs when the pump power exceeds the threshold value P_0^{th}. Its approximate solution growing by exponential law reads $\psi_{p,r}(t,y) = \exp\left(\gamma_{ins}^s t + i\delta\omega_{ins}^s t\right) f_s\left(y/\delta_y\right)$, $\delta_y = \Lambda_y^{1/4}w^{1/2}/\sqrt[4]{\gamma_0}\exp\left(-i\pi/8 - i\arg(\gamma_0/4)\right)$ with $\gamma_{ins}^s = \gamma_0 - (2s+1)\sqrt{|\gamma_0|\Lambda_y/w^2}\sin(\pi/4)$, $s \in \mathbb{Z}$ being the instability growth rate. Setting $\gamma_{ins}^s = 0$ gives the condition for the power threshold P_0^{th}

$$\gamma_0\left(P_0^{th}\right) = (2s+1)\sqrt{\left|\gamma_0\left(P_0^{th}\right)\right|\Lambda_y/w^2}\sin(\pi/4) \tag{6}$$

Then, we solve (5) numerically. Figure 2 shows the temporal dependence of the wave energy in the pump beam localization region in semi-logarithmic scale. The solid curve corresponds to the same dependence but analytically predicted, γ_{ins}^{s}. Being close to each other, they indicate a temporal growth of the EPW amplitude, confirming the excitation of absolute PDI. Figure 3 shows the dependence of the instability growth rate defined on the pump power at the pump beam width of 2 cm and magnetic field ripples $\delta \simeq 0.3\%$ (solid line). The circles are the results of numerical solution of (5). The numerically calculated power thresholds is very low, $P_0^{th} = 287$ kW. Rough analytical estimates of the instability power thresholds in these cases provided by equation (7) overestimate their real values. It should be stressed that the obtained values of the absolute instability power threshold is three orders of magnitude smaller than the value predicted for the induced scattering instability by the standard theory, thus making this PDI inevitable in ITER and leading to the risk of strong anomalous reflection of the power.

4. Conclusions. It is predicted that the electron plasma wave trapping in the edge transport barrier leads to the low power-threshold induced side-scattering absolute parametric decay instability of ordinary microwaves. The minimum power threshold of the PDI leading to scattering at the angle of $0.65\,\pi$ with frequency down-shift of 1.13 GHz is 287 kW. The obtained values of the power-threshold of absolute instability are three orders of magnitude smaller than the value predicted for the induced scattering instability by the standard theory. This nonlinear effect, leading to anomalous reflection of heating power, could easily occur in O1-mode ECRH experiments at ITER, where multiple megawatt pump beams are planned for utilization. Undoubtedly, this effect can have a significant impact on the performance of the ECRH system at ITER and should be taken into account seriously when planning the future experiments.

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