The utility of electron-ion energy coupling: from disruptions to diagnostics

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The extreme mass disparity between electrons and ions inhibits efficient collisional energy exchange between species. As a result, the electron and ion populations reach equilibrium within themselves much faster than they can come to thermal equilibrium with each other. Heat therefore propagates through two distinct but coupled channels, and two temperature modelling is necessary to capture critical features of heat transport in plasmas on time scales comparable to the electron-ion energy exchange rate. Here we present two very different examples that demonstrate the far reaching application of the coupled heat equations: the stabilization of neoclassical tearing modes (NTMs) via the rf condensation effect [1], and perturbative thermal diffusivity measurements [2].

Unstable magnetic islands (NTMs) degrade confinement and play a key role in triggering disruptions. The stabilization of NTMs is accordingly one of the most pressing challenges in fusion research. Although it has long been understood that rf current driven in the center of the island can stabilize NTMs, the nonlinear relationship between the rf deposition and the electron temperature within the island has only recently been explored. Both lower hybrid current drive (LHCD) and electron cyclotron current drive (ECCD) are highly sensitive to changes in electron temperature, since the power deposition is proportional to the number of resonant particles: $P_{dep} \propto n_{res} \propto \exp(-v_{res}^2/v_{th}^2)$. In turn, the island provides a thermally insulated environment, due to the closed magnetic topology and reduced internal thermal diffusivity, such that rf heating within the island produces significant temperature perturbations relative to the separatrix. Then, the rf power deposition will be of the following form:

$$P_{dep} \propto \exp(-\frac{mv_{res}^2}{2(T_{e,0}+\widetilde{T}_e)}) \approx \exp(-\frac{mv_{res}^2}{2T_{e,0}}(1-\frac{\widetilde{T}_e}{T_{e,0}})) \propto \exp(u_e) \tag{1}$$

where $u_e := w_0^2\widetilde{T}_e/T_{e,0}$ is the scaled temperature perturbation, and $W_0 := v_{res}/v_{th}$ is the unperturbed ratio of resonant to thermal velocities.

Since the diffusive timescales of the island are much shorter than the MHD timescales on which the background plasma and island evolve, and adopting a slab geometry for simplicity, the island temperatures are described by the following "steady state" equations:

$$-u_e'' = c(u_i - u_e) + P_0 \exp(u_e) \tag{2}$$

$$-\gamma u_i'' = c(u_e - u_i) \tag{3}$$

where $u_i := w_0^2 \widetilde{T}_i / T_{e,0}$ is the scaled ion temperature perturbation, $c := \tau_{D,e}/\tau_{eq}$ is the scaled equilibration strength, $\tau_{D,e} := 3W_i^2/8\chi_{e,\perp}$ is the characteristic electron diffusion time,$\gamma := \chi_{i,\perp}/\chi_{e,\perp}$ is the ratio of diffusivities, and τ_{eq} is the electron-ion equilibration time. The spatial coordinate has been scaled to the island half width $x_{scl} := W_i/2$, and the solutions are subject to the boundary conditions $u_s(x = \pm 1) = 0$. It has been assumed that whatever ion heating may be present is sufficiently far from the island location, such that $P_i = 0$. The electron and ion temperatures are shown for various coupling strengths in Fig. 1. [3]

In the weakly coupled limit ($c \to 0$), the island electrons experience the full benefit of the rf heating, while the ions remain unheated (relative to the separatrix). In the strongly coupled limit ($c \to \infty$), the effective power is reduced by a factor of $(1+\gamma)$, i.e. $P_0 \to P_0/(1+\gamma)$. It is important to note that the coupling strength c is not set by the energy equilibration rate alone, but by its ratio to the diffusive loss rate ($c = \tau_{D,e}/\tau_{eq}$. Since the coupling strength $c \propto W_i^2$, this means that it will be more efficient to stabilize smaller islands, such that less of the rf power is lost on the ions.

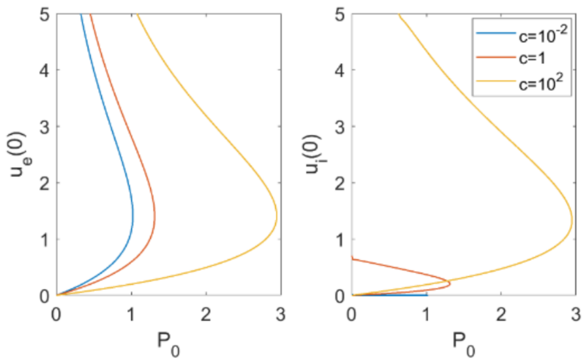

Figure 1: *Electron (left) and ion (right) temperatures at O-point vs. P_0, for $\gamma = 2$*

However, a more robust stabilization technique is possible in a hot ion mode. Hot ion modes, for which $T_i > T_e$, are traditionally of interest due to their superior reactivity and confinement properties. Returning to Eqs. (2)-(3), if $T_{i,0} > T_{e,0}$, there will be an additional heating/cooling term:

$$-u_e'' = c(u_i - u_e + h) + P_0 \exp(u_e) \tag{4}$$

$$-\gamma u_i'' = c(u_e - u_i - h) \tag{5}$$

where $h := T_{i,0}/T_{e,0} - 1$ is the scaled temperature difference at the separatrix. This additional heating gets amplified through the constructive feedback between the electron temperature and the nonlinear rf heating, and creates drastically different outcomes for stabilization, as shown in Fig. 2. For the same rf power, islands can be stabilized at smaller widths, or alternatively, it may cost less power to stabilize islands at a given acceptable width. The details of the stabilization calculation can be found in Ref. [4]. The coupled heat equations therefore reveal an exciting potential synergy between rf stabilization and hot ion mode operation.

The same equations provide surprising insights for the analysis of perturbative thermal diffusivity measurements, in which localized modulated heating is applied to a certain species (usually, and more conveniently, ECH for the electrons) and the phase and amplitude profiles of the resulting heat pulses are used to infer the local thermal diffusivity. Such experiments have long been widely utilized in transport studies, but to date, the most popular analysis method is based on a single fluid model, despite a number of experiments opting for modulation frequencies on the order of the electron-ion equilibration time. A careful analysis of the coupled heat equations not only reveals the inaccuracies of the single fluid-approach, but also suggests a novel measurement technique.

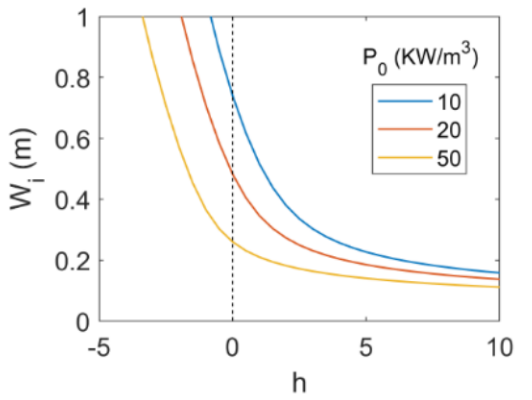

Figure 2: *Island width at stabilization vs. temperature differential h for various P_0.*

The temperature perturbations due to the modulated heating are described by the following equations:

$$\partial_t \widetilde{T}_e - X_e \partial_x^2 \widetilde{T}_e = \nu(\widetilde{T}_i - \widetilde{T}_e) + P\exp(-i\omega t)\delta(x) \tag{6}$$

$$\partial_t \widetilde{T}_i - X_i \partial_x^2 \widetilde{T}_i = \nu(\widetilde{T}_e - \widetilde{T}_i) \tag{7}$$

where a slab geometry has been adopted for simplicity, and $\nu := 1/\tau_{eq}v$ is the e-i energy equilibration rate. Eqs. (6)-(7) approximately hold as long as the length scales of the background are long compared to the wavelength of the heat pulses. The δ-function heat source is appropriate for modelling the highly localized heat deposition characteristic of ECH.

Single fluid treatments correspond to Eq. (6) alone, with $T_i = 0$, in which case the electron diffusivity would be given by the following expression:

$$\chi_{e,app} = \frac{3}{4\phi'(A'/A)} \tag{8}$$

where ϕ and A are the phase and amplitude of the electron temperature perturbations $\widetilde{T}_e$, respectively. The subscript "app" denotes "apparent", as the appropriate two fluid treatment reveals that the quantity obtained with Eq. (8) is often not the true electron diffusivity.

Eqs. (6)-(7) can be easily solved to obtain that the temperature response of both species to modulated heating is a superposition of two modes:

$$\widetilde{T_{e/i}} = [A_{e/i,1}\exp(ik_1|x|) + A_{e/i,2}\exp(ik_2|x|)]\exp(-i\omega t) \tag{9}$$

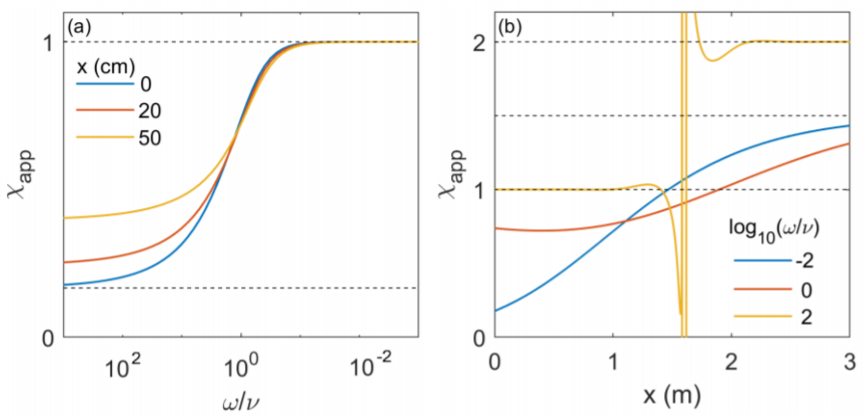

Figure 3: *Apparent diffusivity dependence on space and modulation frequency, for* $\chi_e = 1m^2/s$ *and* $\chi_i = 2m^2/s$

where k_1, k_2 are the two solutions to

$$[1 + (X_e k^2 - i\omega)][1 + (X_i k^2 - i\omega)] = 1 \tag{10}$$

As a result of this superposition, the apparent diffusivity Eq. (8), does not straightfowardly capture the electron diffusivity as intended, but depends on the modulation frequency, the distance from the source, and most importantly, the ion diffusivity. These dependences are summarized in Fig. 3. The single fluid result holds only in the limit of high modulation frequencies, and sufficiently far from the source. Surprisingly, at lower frequencies, the superposition of modes conspire to produce an apparent diffusivity that is lower that the true electron diffusivity.

Not only are these two fluid insights important for the correct interpretation of past experiments, they also suggest the possibility of a novel measurement method. By explicitly recognizing the impact of the ion heat channel on the electron temperature response, and opting for sufficiently slow modulation frequencies, the ion diffusivity may be simultaneously extracted from the same electron temperature measurements that have been taken all along.

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References

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