Advances in the asymptotic description of the electron motion in the strongly radiation-dominated regime

A.S. Samsonov[1], E.N.Nerush[1], I.Yu. Kostyukov[1]

[1] *Institute of Applied Physics of Russian Academy of Sciences, Nizhny Novgorod, Russia*

Introduction

With upcoming laser facilities, such as ELI, investigation of laser-matter interaction in the regime of extreme intensity will become feasible. Theoretical studies of such an interaction are often limited by significantly simplified scenarios due to strong nonlinearity of the interaction, while numeric ones can only provide some phenomenological dependencies, scalings or parameters regions where one or another regime is realized without giving an insight into their nature. Radiation reaction which is an immanent feature of high intensity laser-plasma interaction has been a well-known phenomenon for more than a hundred years, although there is still no general approach to account effect of radiation reaction on the electron dynamics, i.e. solving the following motion equations

$$\frac{d\gamma}{dt} = -\mathbf{vE} - F_{rr}v^2, \tag{1}$$

$$\frac{d\mathbf{v}}{dt} = -\frac{1}{\gamma}\left(\mathbf{E} + \mathbf{v}\times\mathbf{B} - \mathbf{v}\left(\mathbf{vE}\right) - \frac{F_{rr}}{\gamma^2}\right), \tag{2}$$

where $\mathbf{v}$ is the electron velocity normalized to the speed of light c, $\gamma = (1-v^2)^{-1/2}$ is the electron Lorentz-factor, F_{rr} is the radiation-friction force, $\mathbf{E}$ and $\mathbf{B}$ are the electric and the magnetic field respectively normalized to $m_e c\omega/e$, where m_e and $e > 0$ are the electron mass and charge respectively and ω is the characteristic frequency of the EM field.

Previous studies have shown that there is a way to implicitly account for radiation reaction and derive reduced equations of the electron motion in the strongly radiation-dominated regime [1, 2]. The following equations describe a so-called asymptotic or radiation-free trajectory which attracts real electron trajectories

$$\mathbf{E} + \mathbf{v_0}\times\mathbf{B} - \mathbf{v_0}\left(\mathbf{v_0 E}\right) = 0, \tag{3}$$

$$\frac{d\mathbf{r}}{dt} = \mathbf{v_0}\left(\mathbf{E}(\mathbf{r},t), \mathbf{B}(\mathbf{r},t)\right). \tag{4}$$

While this alone can be used to describe dynamics of the electron, this approach is quite limited for two reasons. First, Eqs. (3) – (4) describe real trajectories well only at extremely large intensities $I \gtrsim 10^{25}$ W/cm^2. And second, it does not allow one to find the electron energy and radiation losses while approaching this asymptotic trajectory.

Novel approach

To overcome the problems present in the approach developed in [1, 2], and to describe dynamics of the electron on a time-scale of EM field variation more precisely we introduce a small deviation $\mathbf{v_1} \perp \mathbf{v_0}$ from the asymptotic radiation-free direction, i.e.

$$\mathbf{v} = (1-\delta)\,\mathbf{v_0} + \mathbf{v_1}, \tag{5}$$

where δ can be found from the condition that $|\mathbf{v}| \approx 1$, from which we get $\delta \approx v_1^2/2$. Substituting this into Eqs. (1) – (2), utilizing Eq. (3) and keeping only terms up to the second order, v_1^2, we obtain a set of general equations governing electron dynamics in the strongly radiation-dominated regime

$$\frac{d\mathbf{v_1}}{dt} = -\frac{1}{\gamma}\mathbf{F_1} - \frac{d\mathbf{v_0}}{dt} - \mathbf{v_0}\left(\mathbf{v_1}\frac{d\mathbf{v_0}}{dt}\right), \tag{6}$$

$$\frac{d\gamma}{dt} = -\mathbf{vE} - F_{rr}(\chi), \tag{7}$$

$$\mathbf{F_1} = \left(\mathbf{v_1} - \frac{v_1^2}{2}\mathbf{v_0}\right) \times \mathbf{B} - \mathbf{v_1}\,(\mathbf{v_0E}) - \mathbf{v}\,(\mathbf{v_1E}), \tag{8}$$

$$\chi = \frac{\gamma}{a_S}\,|\mathbf{F_1}|, \qquad F_{rr} \approx \frac{2}{3}\alpha a_S \chi^2, \tag{9}$$

where use of the classical expression for F_{rr} is justified by the fact that χ is proportional to a small term v_1 and thus is also small $\chi \ll 1$.

To demonstrate how Eqs. (6) – (7) can be used in practice, we consider the electron motion in the strongly radiation-dominated regime in two illustrative field configurations: rotating electric field and a plane wave.

Rotating electric field

Let's consider electron dynamics in a uniform rotating electric field, i.e. a field configuration corresponding to the magnetic node of the standing circularly-polarized plane wave. In that case the asymptotic velocity is always directed opposite to the electric field

$$\mathbf{v_0} = -\frac{\mathbf{E}}{E}. \tag{10}$$

It can be assumed that the deviation vector $\mathbf{v_1}$ also rotates with the same frequency as $\mathbf{E}$ and $\mathbf{v_0}$ so we can perform the substitution $d/dt \to \omega\times$, where ω is a unit vector perpendicular to the rotation plane. In that case Eq. (6) takes form

$$\omega \times \mathbf{v_1} + \mathbf{v_0}\,(\mathbf{v_1}\,[\omega \times \mathbf{v_0}]) + \omega \times \mathbf{v_0} + \mathbf{v_1}\frac{E}{\gamma} = 0. \tag{11}$$

Utilizing geometric relations between ω, $\mathbf{v_0}$ and $\mathbf{v_1}$ Eq. (11) reduces to the following relation

$$v_1 = \frac{\gamma}{E} \ll 1. \tag{12}$$

We can find a stationary solution of Eq. (7) corresponding to the case when energy gain in the electric field equals exactly to the radiation losses and obtain the following relation

$$\gamma = \sqrt[4]{\frac{3}{2}\frac{a_S}{\alpha}E}, \tag{13}$$

which combined with Eq. (12) exactly coincides with a well-known Zeldovich solution [3]. A novelty of this result comes from the fact that it is just a special case of a general set of equations (6) – (7).

Plane wave (constant crossed fields)

Some interesting results can be obtained if one applies our asymptotic approach to the electron motion in a plane wave. In a plane wave asymptotic direction coincides with the direction of the Poynting vector

$$\mathbf{v_0} = \frac{\mathbf{E}\times\mathbf{B}}{E^2}. \tag{14}$$

where both $\mathbf{E}$ and $\mathbf{B}$ are functions of the phase $\varphi = x - t$. Note that the fact that a strong wave will always push the electron in the direction of its propagation was recently observed in an exact analytical solution of the electron motion in a plane wave [4].

The term $\mathbf{F_1}$ can be also easily calculated

$$\mathbf{F_1} = \frac{v_1^2}{2}\mathbf{E} - \mathbf{v_1}(\mathbf{v_1}\mathbf{E}), \tag{15}$$

For the sake of simplicity let's assume that initially the deviation vector $\mathbf{v_1}$ is parallel to the electric field, in which case it will stay so at any time instance and thus Eqs. (6) – (7) take form

$$\frac{d\gamma}{dt} = v_1 E - \frac{1}{6}\frac{\alpha}{a_S}\gamma^2 v_1^4 E^2, \tag{16}$$

$$\frac{dv_1}{dt} = -\frac{v_1^2 E}{2\gamma}, \tag{17}$$

$$\frac{d\varphi}{dt} = -\frac{v_1^2}{2}\sqrt{1-\frac{1}{\gamma^2}}. \tag{18}$$

It can be shown that in the strongly radiation-dominated regime (for wave amplitudes $E \gtrsim 1000$) due to decrease of v_1 the phase φ reaches some finite value asymptotically rather than changes indefinitely as in the case without radiation reaction. It means that the electron will eventually

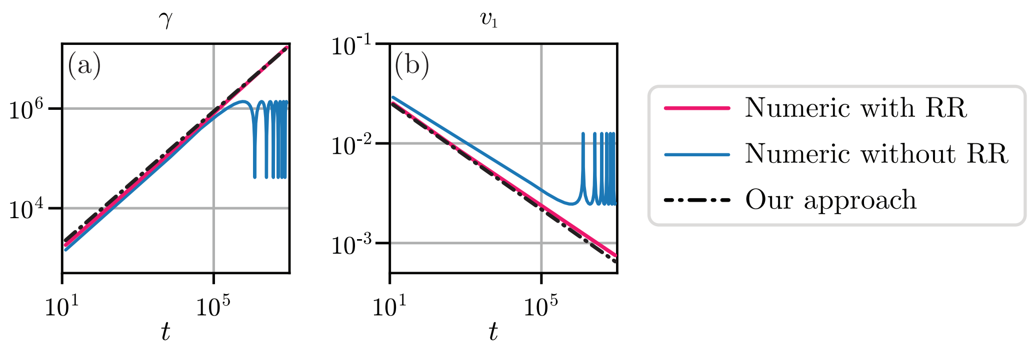

Figure 1: *Electron dynamics in a plane wave with amplitude* $E = 5000$*: (a) Lorentz-factor and (b) transverse velocity* v_1 *as functions of time. Red (blue) lines correspond to the numeric solution of Eqs.* (1) – (2) *with (without) account of radiation reaction, black dashed lines show asymptotics* (19).

only feel constant crossed fields, so E can be assumed to be constant in the equations above. In that case these equations have the following asymptotics

$$\gamma \propto t^{3/5}, \qquad v_1 \propto t^{-2/5}, \qquad \chi \propto t^{-1/5}, \tag{19}$$

which means that solution with account of radiation reaction is not only non-periodic but also features quite an unexpected behavior: instead of slowing down the electron, radiation reaction actually leads to infinite acceleration of the electron. Numerical solution of the Eqs. (1) – (2) confirms that behavior (see Fig. 1).

Conclusion

We have advanced the approach of determining the electron dynamics in the strongly radiation-dominated regime in an arbitrary field configuration originally developed in Refs. [1, 2]. Using improved approach we were able to reproduce a well-known Zeldovich solution and to discover a not yet commonly acknowledged feature of the electron motion in a strong plane wave — unlimited acceleration.

Acknowledgments

The work is supported by Russian Science Foundation Grant No. 20-12-00077.

References

[1] A.S. Samsonov, E.N. Nerush and I.Yu. Kostyukov, Phys. Rev. A **98**, 053858 (2018)

[2] A. Gonoskov and M. Marklund, Phys. Plasmas **25**, 093109 (2018)

[3] Y.B. Zeldovich, Sov. Phys. usp. **18**, 79 (1975)

[4] R. Ekman, T. Heinzl, and A. Ilderton New J. Phys., **23**, 055001 (2021)