Considerations on Monitoring Tearing Modes Stability in Toroidally Rotating MHD Equilibria

E.Lazzaro[1],L.Bonalumi,S.Nowak[1],D.Brunetti[2]

[1] *Institute for Plasma Science and Technology-CNR, Milan, Italy*

[2] *Culham Center of Fusion Energy, Culham (OX), UK*

In tokamak operation the control of dangerous MHD instabilities, possibly in real-time scenarios, must rely on prompt and robust diagnostics of the state and stability of the system. The information on the current J profile details is generally considered undetectable from outside the plasma boundary, but it can be conjectured that it is somehow mapped onto the surface providing a Shannon-Bekenstein information entropy [1], associated to the probability that some internal profile feature (*"mode"*) exists.The set of magnetic signals measured on the outside of the plasma boundary, based on the Zakharov-Shafranov, Shkarofsky,Wootton (ZSSW) [2, 3, 4, 5] current moments has been always used for fast monitoring of key characteristics of the instantaneous equilibrium condition, such as the quantities Δ_{Sh}, the Shafranov centroid shift, β_p, related to the thermal energy content, and internal inductance li related to the current profile peakedness. In addition the fast pick up coils monitor the external magnetic field fluctuations due to internal MHD activity, however without possibility of radial localization of the source. Further ZSSW moments are generally disregarded and their information has not been used to come closer to the assessment, for instance, of the MHD instabilities. Here we explore the potential usefulness of a more complete use of ZSSW moments, in association with the information from fast B perturbation signals, to infer the sign of the tearing modes parameter Δ'. For clarity we set up an analysis of the measurable response to tearing perturbations based on an exact equilibrium model, which is an extension of the Soloveev case with the addition of an equilibrium non uniform plasma rotation $\Omega(\psi)$. Applying the method of inverse equilibrium representation [3], the role of a steady toroidal rotation on the effective value of Δ' is studied in full curvilinear geometry, for the most dangerous (m,n) tearing modes, in presence of toroidal rotation. The relation of selected (externally measurable) ZSSW moments with the calculated stability index, is mapped for different rotation values. The footprint of the stability condition $\Delta' \leq 0$ on some current multipolar moments [2, 3, 4, 5] can then identify stability boundaries, for different rotation conditions. This first discussion on an idealized exact model is proposed for testing the sensitivity of the method in view of application to realistic equilibria, since it relies on few, externally monitorable quantities and very basic assumptions on the tearing modes physics. A simple form of equilibrium is chosen, described by the Grad-Shafranov equation for the poloidal flux function $\psi(R,Z)$:

$$\triangle^*\psi = -R^2 P_0 - R^4\Omega \tag{1}$$

where $P_0 = \mu_0 p_0/\psi_b,\ \Omega(\psi) = \mu_0\rho\omega^2/2\psi_b$ and ρ and ω are the mass density and frequency, respectively. An exact solution of 1 is obtained in the form

$$\psi(R,Z) = c_0 R^2 Z^2 + k(R^2 - R_0^2)^2 + \frac{\Omega(\psi)}{24}R^6 - \psi_{ax} \tag{2}$$

where the coefficients are obtained assigning boundary conditions $\psi = \psi_b = 4kR_0^2 r_b^2$, going through the points $r = r_b,\ \ Z = Z_s = Z(r_b, \pi/2)$ and vanishing on the magnetic axis,with $\psi_{ax} = \frac{\Omega}{24}R_0^6 - \frac{\Omega^2}{256k}R_0^8$. Here $c_0 = (8r_b^2/Z_s^2)k,\ \ k = \frac{Z_s\sqrt{\mu_0 p_0}}{4R_0 r_b\sqrt{2(Z_s^2+r_b^2)}}$. The poloidal B_θ field in rectified flux coordinates (r,θ,ϕ) is $B_\theta(r) = \frac{\psi'(r)}{\sqrt{g}}$ and $B_\phi = \frac{T}{R_0}$ and $\sqrt{g}$ is the Jacobian of the transformation from the coordinates (R,ϕ,Z). A simple but crucial observation should be made on the structure of the current density on the r.h.s. of eq.1. It is basically an expression of the fundamental force balance in *toroidal geometry*, and is strictly related to the geometric and global properties of the equilibrium configuration, which are associated with "moments" measured outside the plasma; toroidicity, shaping and diamagnetic measurements help removing certain degeneracies, allowing for instance separation of β_p and ℓ_i [2, 3, 5, 7]. Here we advance the conjecture that this "irreducible" toroidal effect may carry also other information, so far disregarded, related to certain stability conditions. We apply the method of inverse equilibrium representation [6] in presence of steady toroidal rotation to study the effective value of Δ' in full curvilinear geometry, for the most dangerous (classical) (m,n) tearing modes. The Zakharov-Shafranov multipole moments of the toroidal current density J_ϕ are given by weighted contour integrals of the peripheral magnetic field components tangent and normal to closed paths,surrounding the plasma:

$$Y_m = \frac{1}{\mu_0 I}\int J_\phi f_m dS_\phi = \frac{1}{\mu_0 I}\oint f_m B_\theta(r_b)d\ell = \frac{1}{\mu_0 I}\int_0^{2\pi} f_m B_\theta(r_b)\sqrt{g_{\theta\theta}(r_b)}\,d\theta \tag{3}$$

For the sake of argument the integration contour is the plasma boundary,at $r = r_b$. The weighing functions f_m are solutions of the Grad-Shafranov equation in vacuum [2, 3, 4, 5], transformed,for convenience, to be vanishing on the magnetic axis [4, 9]. The tearing mode equation for the first order perturbed helical poloidal flux $\widetilde{\psi}_{m,n}$, in curvilinear (toroidal) geometry takes the form:

$$\langle\frac{g_{\theta\theta}}{\sqrt{g}}\rangle\frac{\partial^2\widetilde{\psi}_{m,n}}{\partial r^2} + \langle\frac{g_{\theta\theta}}{\sqrt{g}}\rangle'\frac{\partial\widetilde{\psi}_{m,n}}{\partial r} - \left[m^2\langle\frac{g_{rr}}{\sqrt{g}}\rangle + \frac{m}{m-nq}\langle J^*\rangle'\right]\widetilde{\psi}_{m,n} = 0 \tag{4}$$

Near the rational surface $r = r_s$ the strength of driving term of the tearing perturbation in eq.4 can be explicited as $J^{\star'}\delta_\eta = -AR^2 - BR^4$ where the equilibrium current used here is consistent with eq.1, and $A = \frac{4r_s\sqrt{k}}{T\sqrt{2c_0}}$ and $B = \frac{\Omega}{P_0}A$; δ_η indicates the width of the reconnecting layer. By considering the scaling of the f_m functions (see [9]) it can be expected that a ZSSW moment sensitive to Δ' could be assembled by some linear combination of f_m.The choice is *not unique*, but here the simplest combination $f_\Delta \approx \alpha f_1 + \beta f_3$, is tested. By matching the terms with corresponding powers of R, the constants are determined as $\alpha = -2R_0\frac{P_0}{T},\ \ \beta = -4R_0^2\frac{\Omega}{T}$ leading eventually to the *practical* expressionn

$$Y_\Delta = -\frac{2R_0\psi'(r_b)}{\mu_0 I}\oint [\frac{P_0}{T}f_1 + 2R_0\frac{\Omega}{T}f_3]\frac{\sqrt{g_{\theta\theta}(r_b)}}{\sqrt{g(r_b)}}d\theta \tag{5}$$

The tests presented here are done for $k = 0.007$,$c_0 = 0.05$, $R_0 = 1.9$, $r_b = 1$, $\Omega = 0.007$, $p_0 = 0.1$.

The relation of Δ' with $\Delta Y_\Delta = Y_\Delta - Y_{\Delta=0}$, for modes $m = 2, n = 1$ and $m = 3, n = 2$ is shown in (Fig. 1) and (Fig. 2) for different values of the toroidal rotation Ω. It is apparent that $\Delta Y_\Delta \geq 0$ is associated with $\Delta' \leq 0$, so the change of sign of ΔY_Δ makes this signal very suitable to monitor the (linear) stability condition, in presence or absence of rotation.

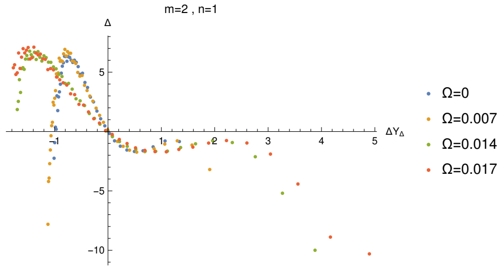

Figure 1: Δ'*vs.* ΔY_Δ *for mode m=2,n=1, for* $0 \leq \Omega \leq 0.017$*;* $\Delta Y_\Delta \geq 0$ *is associated with* $\Delta' \leq 0$*.*

A summary of the results is represented in stability domains in the parameters space (c_0, k, T)(Fig. 3), (Fig. 4). The parameter T modifies only the safety factor profile q, while the geometry of the system is kept fixed.

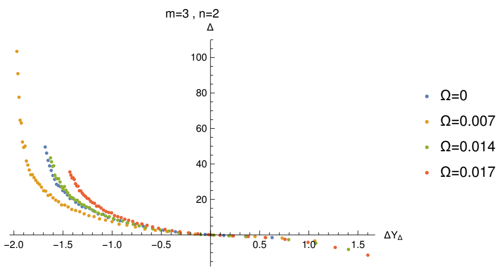

Figure 2: Δ'*vs.* ΔY_Δ *for mode m=3,n=2, for* $0 \leq \Omega \leq 0.017$.

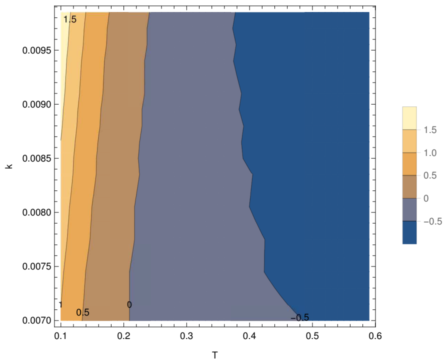

Figure 3: (k, T) *domain of stability* $\Delta Y_\Delta \geq 0$*, for* $m = 2, n = 1$.

The example proves constructively that weighted combinations of moments from the external magnetic signals, can retrieve promptly information mapped from the interior to the boundary and relevant to the linear stability of tearing perturbations.

Conclusions

In analogy with a "holographic principle",known in other branches of physics [1], which states that all information in a volume of space is stored on two dimensional bounding surface, an *extension* of a long established technique [2, 3, 4] is suggested for the inverse problem of detection of information on internal details of tokamak current profile. The detectability of the sign of the tearing mode stability index Δ' is shown to be possible from combinations of *externally measurable* magnetic multipole moments. This first particular constructive test is meant to suggest applications to more general cases.

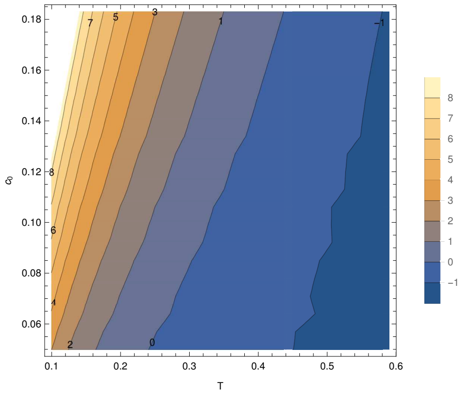

Figure 4: (c_0, T) *domain of stability* $\Delta Y_\Delta \geq 0$*, for* $m = 2, n = 1$.

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