Anomalous absorption at ECRH and ways to reduce it

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The electron cyclotron resonance heating (ECRH) is a popular method to produce fusion relevant plasma in magnetic confinement devices. It is based on theoretical predictions of a localized microwave energy deposition and of suppression of nonlinear phenomena which can accompany the microwave propagation and damping. However, during the last decade various anomalous effects (anomalous microwave scattering producing strong spurious radiation interfering with ECE and CTS diagnostics, ion acceleration, evident broadening of the ECRH power deposition profile and gyrotron frequency sub-harmonics emission) were discovered in the ECRH experiments at different toroidal devices. They were interpreted as a consequence of low-power-threshold absolute parametric decay instabilities (PDIs) excited in the presence of a non-monotonic (hollow) density profile often encountered at ECRH. The most dangerous scenario discovered quite recently [1-3] is a PDI leading to excitation of two upper hybrid (UH) daughter waves at least one of which is trapped along the direction of the plasma inhomogeneity and localized on a magnetic surface due to the finite pump width. The developed theoretical model [1] allowed explaining the anomalous backscattering effect first observed in the X2 ECRH experiments at TEXTOR [4] and anomalous emission at half the pump wave frequency predicted in [3] and first seen at ASDEX-UG [5]. A substantial anomalous absorption, which can lead to broadening of the power deposition profile, was also predicted by the model [1, 2] and demonstrated in the model experiment [6].

In this paper the main stress is put on the analysis of the ways to decrease the influence of anomalous phenomena on the efficiency and locality of the power absorption and on operation of microwave diagnostics. Suppressing the low-power-threshold PDIs and reducing of the related anomalous absorption rate by variation of the beam width, increasing of a single microwave beam power are discussed within the developed theoretical model [1-3]. The general case of only one trapped UH wave is considered. We suppose the local maximum of a non-monotonic UH frequency profile to lie close to the origin of the coordinate system and consider the pump extraordinary (X) wave propagating quasi-perpendicularly to the magnetic field in the plasma density inhomogeneity *x*-direction

$$\mathbf{E}_0 = \mathbf{e}_0 \sqrt{\frac{8P_0}{cw^2}} \sqrt{\frac{c}{\upsilon_{0g}(x)}} \exp\left(i\int^x k_x(\omega_0, x')\,dx' - i\omega_0 t - \frac{y^2}{2w^2} - \frac{z^2}{2w^2} \right) + c.c. \quad (1)$$

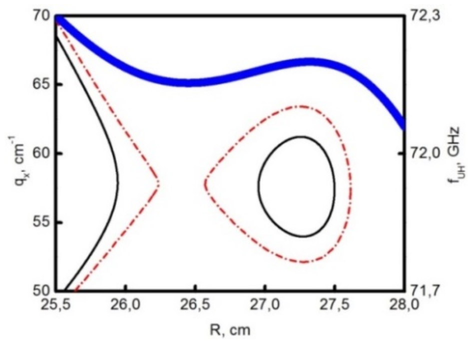

Figure 1. The dispersion curves of the trapped UH waves ($f_m = 70.6$ GHz , solid line) and ($f_n = 70.58$ GHz , dashed-dotted line). The UH frequency profile given by the thick solid line. $T_e = 600$ eV , $f_{ce} = 51.72$ GHz .

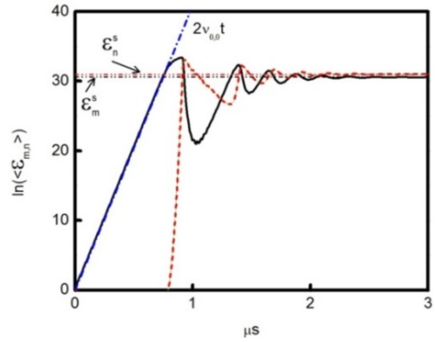

Figure 2. Temporal evolution of primary (solid line) and secondary (dashed line) plasmons energies within the spot of a beam given in a semi-logarithmic scale. The dashed dotted line – the amplification factor with the analytical growth rate (8) whereas the dash-double dotted lines represent (9). $w = 1$cm, $P_0 = 600$kW .

where $\mathbf{e}_0 = \mathbf{e}_y - i\mathbf{e}_x g_0 / \varepsilon_0$ is the polarization vector with $\mathbf{e}_x$ and $\mathbf{e}_y$ being the unit vectors in the corresponding directions, $k_x = \omega_0 / c\sqrt{\varepsilon_0 - g_0^2 / \varepsilon_0}$ stands for the local wavenumber, υ_{0g} implies the group velocity, $g,\varepsilon_0 = g,\varepsilon(\omega_0)$ are the perpendicular components of the cold-plasma dielectric tensor. The UH decay waves are described by their potentials

$$\varphi_a = \frac{C_a(y,z)}{2}\phi_m(x)\exp(-i\omega_m t) + c.c., \varphi_b = \frac{C_b(\mathbf{r})}{2}\exp\left(i\int^x q_x^-(\omega_0-\omega_m,x')dx' + i(\omega_0-\omega_m)t\right) + c.c. \qquad (2)$$

The eigenfunction $\phi_m(x)$ describes the localized UH wave. We give its explicit representation without the detailed derivation (see [7])

$$\phi_m = L_m^+(x)^{-1/2}\exp\left(i\int_{x_l^*}^x q_x^+(\omega_m,\xi)d\xi - i\frac{\pi}{4}\right) + L_m^-(x)^{-1/2}\exp\left(i\int_{x_l^*}^x q_x^-(\omega_m,\xi)d\xi + i\frac{\pi}{4}\right)$$
$$L_m^\pm(x) = \left|D_q\left(q_x^\pm(x)\right)\right|\int_{x_l^*}^{x_r^*}d\xi\left(\left|D_q^+(\xi)\right|^{-1} + \left|D_q^-(\xi)\right|^{-1}\right) \qquad (3)$$

In (2) and (3) the wavenumbers $q_x^\pm(\xi) = q_x^\pm(\omega,\xi)$ are the solutions of the local UH dispersion equations $D_{UH}(\omega_m, q_x^\pm) = 0$ and $D_{UH}(\omega_0-\omega_m, q_x^\pm) = 0$ obtained at $q_z = 0$, $D_q^\pm = \partial D_{UH} / \partial q_x|_{q_{1x}^\pm(x)}$ and the localized wave's frequency obeys the quantization condition $\int_{x_1^*}^{x_r^*} q_x^+(\xi)d\xi + \int_{x_r^*}^{x_1^*} q_x^-(\xi)d\xi = \pi(2m+1)$ with $x_{l,r}^*$ being the turning points of the trapped UH wave. The amplitude of the second UH wave excited due to the non-linear interaction of the pump and the trapped UH wave and running out of the decay region is given by equation

$$\frac{\partial C_b}{\partial x} = -iC_a\sqrt{\frac{8P_0}{cw^2}}\frac{\chi_e^{(pr)}}{2H_0}\frac{\phi_m}{D_q^-(\omega_0-\omega_m,x)}\exp\left(-i\int^x\left(q_x(x') + k_0(x')\right)dx'\right) \qquad (4)$$

where $\chi_e^{(pr)}$ is the second-order plasma susceptibility [8] describing the nonlinear coupling of the pump wave and two UH waves. Integrating (4) with the boundary condition $C_b\big|_\infty = 0$ and substituting it into the equation describing the trapped UH wave amplitude we obtain equation

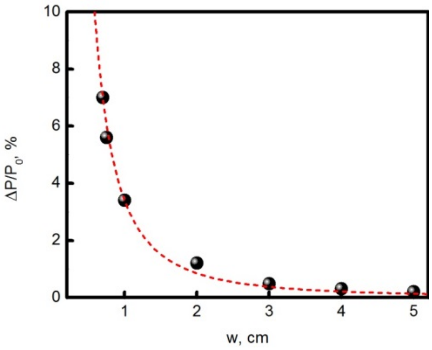

Figure 3. The dependence of the anomalous absorption rate on the pump beam's radius w. The closed circles are the result of a numerical solving. The dashed line gives the analytical prediction (10) and $P_0 = 1$ MW .

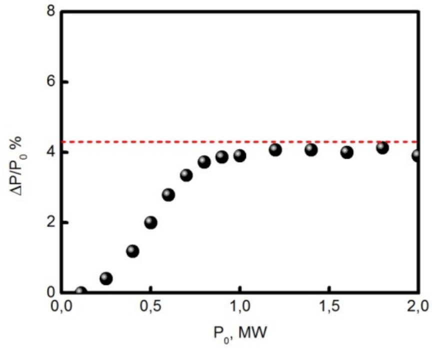

Figure 4. The dependence of the anomalous absorption rate on the power of the pump. The closed circles are the result of a numerical solving. The dashed line is the prediction (10), $w = 1$ cm .

describing the primary decay instability [7]. The dispersion curves for the UH waves taking part in the primary PDI and its saturation are shown in Fig.1 for TEXTOR parameters. As it was shown in [9], a cascade of consequent UH wave decays into secondary UH wave and ion Bernstein wave provides the effective mechanism for the PDI saturation. Assuming weak pump wave depletion typical for the case of odd number of secondary decays [9] we obtain the following equations describing both exponential growth of normalized UH wave amplitudes and the PDI saturation

$$\begin{aligned} &\frac{\partial a_m}{\partial t} - i\Lambda_{my}\frac{\partial^2 a_m}{\partial y^2} - i\Lambda_{mz}\frac{\partial^2 a_m}{\partial z^2} = \gamma_p a_m \exp\left(-\frac{y^2}{w^2} - \frac{z^2}{w^2}\right) - \gamma_s |a_n|^2 a_m \\ &\frac{\partial a_n}{\partial t} + i\Lambda_{ny}\frac{\partial^2 a_n}{\partial y^2} + i\Lambda_{nz}\frac{\partial^2 a_n}{\partial z^2} = \gamma_s |a_m|^2 a_n \end{aligned} \qquad (5)$$

where $\Lambda_{ly,z} = <\partial^2 D_{UH} / 2\partial q^2_{y,z}> / <D_{l\omega}>$, $<D_{l\omega}> = <\left|\partial D_{UH} / \partial\omega\right|_{\omega_l}>$, $l=m,n$ are the coefficients averaged over the corresponding UH wave localization area and describing its diffraction energy loss along the magnetic field. We have also introduced in (5) the coefficients determining the primary and secondary decay daughter wave growth [10]

$$\gamma_p = \frac{\left|\chi_e^{(nl,pr)}\right|^2 l_{dp}^2}{\left|L_m^-\right|\left|D_q^-\left(\omega_0-\omega_m\right)\right| < D_{m\omega} >} \frac{2P_0}{\upsilon_{0g} w^2 H_0^2}\Bigg|_{x_{dp}} \sim \frac{P_0}{w^2} \qquad (6)$$

$$\gamma_s = \frac{4|e|^2}{\sqrt{\omega_m\omega_n}T_e} \frac{\left|\chi_e^{(s)}\right|^2}{\left|L_m^+\right|\left|L_n^+\right| < D_{m\omega} >< D_{n\omega} > \left|D_{Iq}^+\right|} \frac{l_{ds}^2}{w^2}\Bigg|_{x_{ds}} \sim \frac{1}{w^2} \qquad (7)$$

where $l_{dp.s} = \left(\left|d^2\Delta K_{p,s} / dx^2\right| / 6\right)^{-1/3}_{x_{dp,s}}$, $\Delta K_p = q^-_{mx} - q^-_x\left(\omega_0-\omega_m\right) - k_x$, $\Delta K_s = q_{mx} - q_{nx} - q_{Ix}$, $\chi_e^{(s)}$ is the non-linear susceptibility describing the coupling of electrostatic waves [8] and

$D_{Iq}(\omega_{IB}) = \partial D_{IB} / \partial q|_{q_{IB},\omega_{IB}}$ with q_{IB} being a solution of the IB wave dispersion relation $D_{IB} = q_{IB}^2 + \chi_i^{(l)}(q_{IB},\omega_{IB}) + \chi_e^{(l)}(q_{IB},\omega_{IB}) = 0$ in which $\omega_{IB} = \omega_m - \omega_n$ and $\chi_i^{(l)}(q_{IB},\omega_{IB})$ is the linear ion susceptibility. The primary instability described by the first equation in (5) starts to develop if the pump power exceeds its threshold value P_0^{th}. If the pump power significantly exceeds the threshold value $P_0 >> P_0^{th}$ the growth-rate is determined by the following equation [10]

$$\nu_{k,l} = \gamma_p - (2k+1)\sqrt{|\gamma_p|\Lambda_{my}/2w^2} - (2l+1)\sqrt{|\gamma_p|\Lambda_{mz}/2w^2} \sim 1/w^2,\ k,l \in \mathbb{Z} \qquad (8)$$

The saturation levels of UH waves energy density ($\varepsilon_{m,n}^s = |a_{m,n}^s|^2$) can be estimated as follows

$$\varepsilon_m^s \approx 1/\tau_n |\gamma_s| \propto 1, \quad \varepsilon_n^s \simeq |\gamma_p|/|\gamma_s| \propto P_0 \qquad (9)$$

where $\tau_n = \min(w^2/\Lambda_{my}, w^2/\Lambda_{mz}) \simeq w^2/\Lambda_{mz}$ stands for the diffraction losses time. We describe the weak pump wave depletion using the perturbation procedure. In this approximation the variation of the pump wave energy flux is $\delta S_x \simeq 4\gamma_p T_e/(\pi w^2)|a_m(y,z)|^2 \exp(-(y^2+z^2)/2w^2)$. Substituting (9) in the last equation, we finally arrive at

$$\Delta P / P_0 \approx 8\gamma_p T_e \varepsilon_m^s / P_0 \propto 1/w^2 \qquad (10)$$

Behavior of the UH eigenmode amplitudes governed by (5) is shown in Fig. 2 by solid (primary wave) and dashed (secondary wave) curves. The dashed-double-dotted horizontal lines there stand for a rough analysis predictions for the stationary regime (9). The anomalous absorption rate dependences on the pump beam radius and power predicted in the framework of equations (5) are shown in Fig.3 and Fig.4 accordingly. Summarizing the results of the paper, it can be concluded that the microwave pump anomalous absorption rate caused by a low power-threshold PDI, in which only one trapped UH wave is excited, can be reduced by increasing the pump beam radius, but not power.

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1. E. Z. Gusakov et al. *Physics of Plasmas* **23**, 082503 (2016).
2. E. Z. Gusakov et al. *Nucl. Fusion* **59**, 104003 (2019).
3. E. Z. Gusakov et al. Nucl. Fusion **59**, 106040 (2019).
4. S.K. Nielsen et al. *Plasma Phys. Control. Fusion* **55**, 115003 (2013).
5. S. K. Hansen et al. *Proc. of 46th EPS Conference on Plasma Physics* P 1.1075
6. A. B. Altukhov et al. *Europhys. Lett.* **126**, 15002 (2019).
7. A Yu Popov et al. *Europhys. Lett.* **116**, 45002 (2016).
8. E. Z. Gusakov et al. *Plasma Phys. Control. Fusion* **61**, 085008 (2019).
9. E. Z. Gusakov et al. *Physics of Plasmas* **25**, 062106 (2018).
10 E. Z. Gusakov et al. *Nucl. Fusion* **60**, 076018 (2020).