Fast Prediction of Transport near the Critical Gradient

M.J. Pueschel[1,2], P.-Y. Li[3], and P.W. Terry[3]

[1]*Dutch Institute for Fundamental Energy Research, 5612 AJ Eindhoven, The Netherlands*

[2]*Eindhoven University of Technology, 5600 MB Eindhoven, The Netherlands*

[3]*University of Wisconsin-Madison, Madison, Wisconsin 53706, U.S.A.*

In magnetic-confinement fusion research, one of the fundamental questions can be phrased as *for a given set of profile parameters and magnetic geometry, what rate of turbulent transport is predicted?* Two different classes of tools are commonly used to answer this question for specific scenarios. Nonlinear gyrokinetic [1] codes base their results on a high-fidelity framework at the expense of significant computational intensity. By contrast, quasilinear gyrofluid-based transport solvers [2, 3] aim to deliver answers of reasonable accuracy on a much shorter time scale, but may fail to predict fluxes correctly in certain parameter regimes. One particularly important such regime is the range of driving pressure gradients where linear modes are unstable but very little turbulent flux is observed, also referred to as the Dimits regime [4]. Throughout a major fraction of the minor radius, fusion experiments are operating close to the Dimits regime, and improving transport predictions from fast reduced solvers will make a key contribution on the path to fusion energy.

Recent advances in saturation theory [5] have employed a saturation efficiency factor, the so-called triplet correlation time τ, that captures energy transfer between triplets of wavenumbers, as it arises in the Vlasov advective nonlinearity. In the important case of zonal-flow-mediated turbulence [6], the most prominent triplet interaction tends to occur between the dominant instability (streamer) at wavenumber $k = (0, k_y)$, a zonal flow at $k' = (k'_x, 0)$, and a third mode (sideband) at $k'' = k - k'$; here, x is the radial and y the toroidal direction. If one further assumes a weak-turbulence limit where nonlinear modifications to linear complex wave frequencies ω_c are negligible, and relying on a zero-frequency assumption for the zonal flow, the triplet correlation time becomes

$$\tau = \frac{1}{i\omega_\mathrm{c}(k'') - i\omega_\mathrm{c}^*(k)} \ . \tag{1}$$

As detailed in Ref. [5], this quantity modifies the quasilinear heat flux per

$$Q_s^\mathrm{QL} = \omega_{Ts} \sum_{j,k} \mathscr{C}_k \frac{\gamma_{j,k} w_{j,k}}{\langle k_\perp^2 \rangle} \frac{1}{\mathrm{Re}(\tau_{j,k})} \ , \tag{2}$$

where $s = \mathrm{i}, \mathrm{e}$ denotes particle species (ions/electrons), ω_T is the normalized temperature gradient, indices j and k cover all unstable eigenmodes at a given wavenumber and all wavenumbers k_y, respectively, $\mathscr{C}_k$ is a model constant that serves as a spectral shape in k_y, matching the nonlinear flux spectrum at the reference parameters, γ is the linear growth rate, w is a weight factor accounting for the complex phase in the transport definition, and $\langle k_\perp^2 \rangle$ is the field-line-averaged squared perpendicular wavenumber weighted with the square electrostatic potential of the eigenmode.

This model has been deployed successfully [7, 8], significantly improving quasilinear predictions – compared with models lacking τ – for systems experiencing nonlinear electromagnetic stabilization [9, 10, 11]. More recently, it has been applied to the Dimits regime [12].

In Fig. 1, Q^{QL} from different quasilinear models is shown for a scenario of trapped-electron-mode (TEM) turbulence [13] in the Madison Symmetric Torus reversed-field pinch [14]. The mismatch between the standard quasilinear model (red diamonds) and the nonlinear fluxes (dashed black line) represents the Dimits regime. When the streamer and sideband modes enter a strong resonance as the critical gradient for instability is approached, $\mathrm{Re}(\tau)^{-1}$ and thus quasilinear fluxes become small, accounting for a majority of the transport reduction in the Dimits regime.

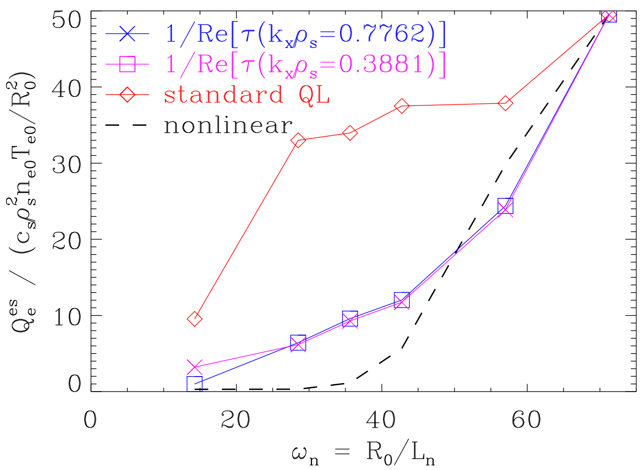

Figure 1: *Quasilinear flux models with (crosses/squares) and without (diamonds) the τ factor, compared with nonlinear simulations (dashed) for a reversed-field-pinch TEM turbulence scenario scanning over the normalized density gradient ω_n. Only when including τ can the flux reduction in the Dimits regime be recovered.*

The situation for a standard ion-temperature-gradient-driven (ITG) turbulence case [4] is visually similar, as is apparent from Fig. 2. However, while τ is behaving qualitatively the same, it contributes less to transport reduction than does the effect of *sideband stabilization*. This phrase refers to the sideband mode becoming linearly stable, in which case energy is removed from the turbulence at a much faster rate, leading to only minimal contributions to transport at the affected wavenumbers. Notably, the results are found to be insensitive to the choice of the zonal-flow k_x as long as $k_x \geq 0.1$.

The results shown here not only highlight the physical underpinnings of the Dimits regime but also confirm that it is possible to recover with reasonable accuracy heat fluxes in this regime by means of quasilinear modeling. Work is presently ongoing to study in which situations a more elaborate saturation theory [15] may be required that accounts not only for τ but also for possible variation in the nonlinear coupling coefficients.

Lastly, it is important to consider how this improved model may be implemented in existing quasilinear transport solvers. First, a more general treatment of the zonal-flow wavenumber k_x is desirable, where a sum over different k_x is employed that is weighted by, e.g., the radial electric field spectrum or the nonlinear energy transfer spectrum at the nonlinear reference point. Second, codes have to possess the capability to obtain eigenvalues for sideband modes centered at finite radial wavenumbers k_x. This is presently the case in the TGLF framework [2] but not yet in QuaLiKiz [3].

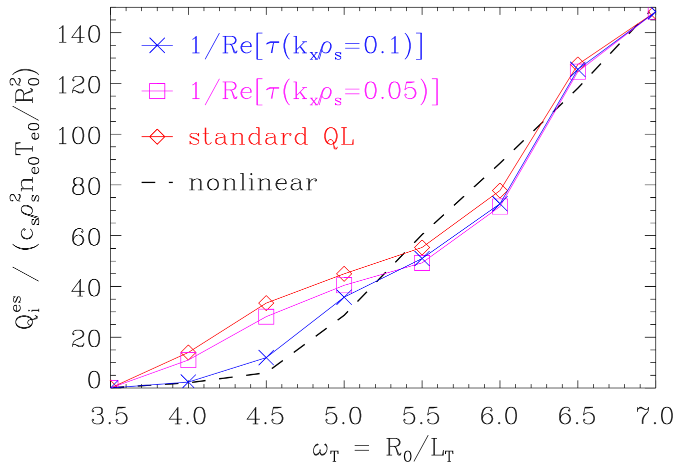

Figure 2: *Quasilinear flux models with (crosses/squares) and without (diamonds) the τ factor, compared with nonlinear simulations (dashed) for a standard ITG turbulence scenario scanning over the normalized ion temperature gradient ω_{Ti}. Only when including τ and the effect of sideband stabilization can the flux reduction in the Dimits regime be recovered.*

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