# Laser-induced Cherenkov radiation in a warm two-fluid model

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***Abstract.*** As an application of the general, warm two-fluid description, we consider a laser exciting a perpendicularly magnetised ion-electron plasma. In this setup, the laser can couple to the plasma's M mode to emit Cherenkov radiation. We show that this emission is restricted to a forward cone, whose angle $\theta$ is limited by the electron thermal velocity $v_\mathrm{e}$ as $\theta = \arccos v_\mathrm{e}/c$, where $c$ denotes the light speed.

## Introduction

Cherenkov radiation can occur whenever a charged particle travels through a medium with a refractive index $n$ larger than 1. However, it can also occur for photon bunches propagating in a plasma, as verified experimentally by Yugami et al. (2002). As an application of our recent work regarding the two-fluid description of a warm, ion-electron plasma (De Jonghe and Keppens, 2020, 2021), we here consider a laser pulse interacting with a warm, ion-electron plasma. Using our new and unambiguous plasma wave labelling scheme (De Jonghe and Keppens, 2020), the coupling is with the modified electrostatic (M) mode. As we will show, the laser-pulsed Cherenkov emission in a warm plasma is restricted to a cone around the laser pulse, contrary to the cold plasma case, where radiation can theoretically be emitted at any forward angle.

## Model and setup

Assuming a homogeneous background of ions and electrons at rest in a uniform magnetic field $\mathbf{B}$, as well as charge neutrality and plane wave solutions ($\sim \exp[i(\mathbf{k}\cdot\mathbf{x}-\omega t)]$) in the continuity, momentum, and energy, complemented by Maxwell's equations, the result is a polynomial dispersion relation of sixth order in the squared frequency $\omega^2$ and of fourth order in the squared wavenumber $k^2$ ($k = |\mathbf{k}|$), as demonstrated in Goedbloed et al. (2019). The coefficients of this dispersion relation depend on the electron and ion cyclotron frequencies, $\Omega_\mathrm{e} = eB/m_\mathrm{e}$ and $\Omega_\mathrm{i} = ZeB/m_\mathrm{i}$, the electron and ion thermal velocities, $v_\mathrm{e} = \sqrt{\gamma p_\mathrm{e}/n_\mathrm{e} m_\mathrm{e}}$ and $v_\mathrm{i} = \sqrt{\gamma p_\mathrm{i}/n_\mathrm{i} m_\mathrm{i}}$, and the propagation angle $\theta$ between the wavevector $\mathbf{k}$ and the magnetic field $\mathbf{B}$. Here, $B = |\mathbf{B}|$, $e$ is the fundamental charge, $Z$ the ion charge number, and $\gamma = 5/3$ the ratio of specific heats. $m_\alpha$, $n_\alpha$, and $p_\alpha$ are the mass, number density, and pressure, respectively, for electrons ($\alpha = \mathrm{e}$) and ions ($\alpha = \mathrm{i}$). Writing $\mu = Zm_\mathrm{e}/m_\mathrm{i}$, all quantities are normalised using the plasma frequency

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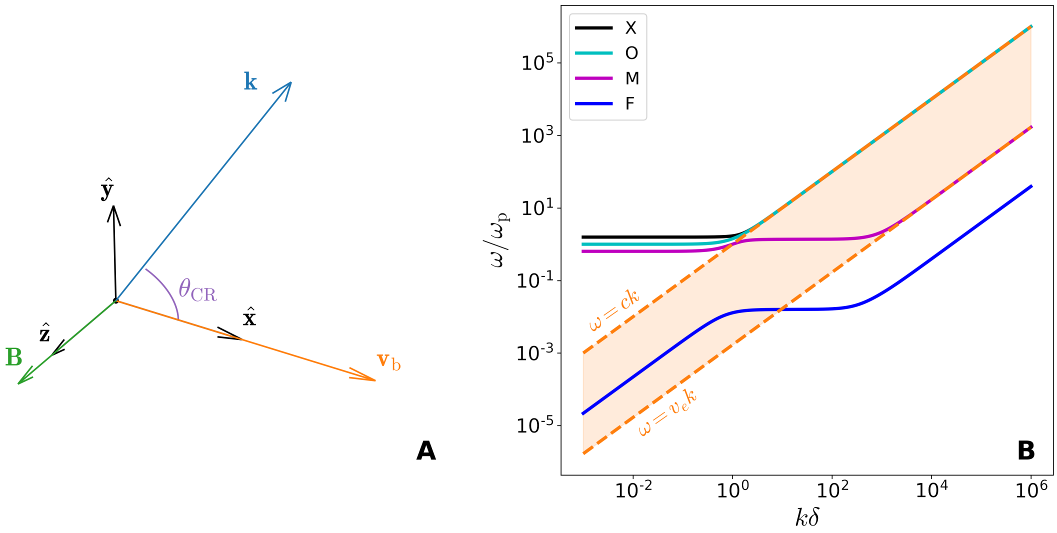

Figure 1: (A) The laser beam propagates with velocity $\mathbf{v}_\mathrm{b}$ perpendicular to the magnetic field $\mathbf{B}$. The Cherenkov radiation with propagation vector $\mathbf{k}$ is emitted at an angle $\theta_\mathrm{CR}$ with respect to the laser beam. (B) Two-fluid dispersion diagram of perpendicular propagation for typical magnetosphere parameters ($\mu = 1/1836$, $E = 0.935$, $v = 1.68 \times 10^{-3}$, $w = 3.92 \times 10^{-5}$, from Goedbloed et al. (2019)). The orange area visualises all angles at which the laser can couple to the M mode through the relation $\omega = (v_\mathrm{b} \cos\theta_\mathrm{CR})k$.

$\omega_\mathrm{p} = \sqrt{(1+\mu)e^2 n_\mathrm{e}/\varepsilon_0 m_\mathrm{e}}$, with $\varepsilon_0$ the vacuum permittivity, and the light speed $c$ as $\bar{\omega} = \omega/\omega_\mathrm{p}$, $E = \Omega_\mathrm{e}/\omega_\mathrm{p}$, $I = \Omega_\mathrm{i}/\omega_\mathrm{p}$ $(= \mu E)$, $v = v_\mathrm{e}/c$, $w = v_\mathrm{i}/c$, and $\bar{k} = k\delta$, with $\delta = c/\omega_\mathrm{p}$ the skin depth. A thorough analysis of the dispersion relation can be found in De Jonghe and Keppens (2020), as well as some applications to whistler modes in the Earth's magnetosphere in De Jonghe and Keppens (2021).

Here, we consider a laser beam exciting a plasma wave in this warm ion-electron plasma, as described by Yoshii et al. (1997); Muggli et al. (1999). The laser beam propagates perpendicular to the background magnetic field $\mathbf{B}$, say $\mathbf{B} = B\,\hat{\mathbf{z}}$ and $\mathbf{k}_\mathrm{laser} = k_\mathrm{laser}\,\hat{\mathbf{x}}$, as shown in Fig. 1A.

## Cherenkov radiation angle

In this setup, the phase speed of the laser pulse, which is approximately $c$, can exceed the modified electrostatic (M) mode's phase speed. When this occurs, the laser and the M mode will couple through the emission of Cherenkov radiation (CR). For a first consideration, we limit ourselves to emission perpendicular to the magnetic field (i.e. in the $xy$-plane). In this case, only 4 modes (F, M, O, X) are left from the full sixtet since slow and Alfvén branches do not propagate perpendicular to $\mathbf{B}$. This case is illustrated in Fig. 1B, where the dashed orange $\omega = ck$ line exceeds the purple M mode line for larger values of $k$. The angle $\theta_\mathrm{CR}$ in the $xy$-plane with respect to the laser beam at which this radiation is emitted, depends on the refractive index $n_\mathrm{M}(\omega) = ck_\mathrm{M}(\omega)/\omega$ as

$$\cos\theta_\mathrm{CR} = \frac{1}{\beta_\mathrm{ph} n_\mathrm{M}(\omega)}, \tag{1}$$

where $\beta_\mathrm{ph} = v_\mathrm{ph}/c \simeq 1$ is the normalised phase speed of the laser beam (Buts et al., 2006). As can be seen in eq. (1), the emission angle depends on the frequency, and thus different frequencies are emitted at different angles.

In a cold plasma the M mode's frequency at perpendicular propagation is confined to the interval between the lower cutoff frequency

$$\omega_\mathrm{l} = \left[1 + \frac{1}{2}(E^2 + I^2) - \frac{1}{2}|E - I|\sqrt{(E+I)^2 + 4}\right]^{1/2} \omega_\mathrm{p}\ , \tag{2}$$

and the upper hybrid frequency

$$\omega_\mathrm{UH} = \left[\frac{1}{2}\left(1 + E^2 + I^2 + \sqrt{(1 + E^2 + I^2)^2 - 4EI(1 + EI)}\right)\right]^{1/2} \omega_\mathrm{p}\ , \tag{3}$$

for all wavenumbers. Since the wavenumber can take any positive value, the refractive index $n_\mathrm{M} = ck/\omega_\mathrm{M}(k)$ can then also take any positive value and thus radiation can be emitted at all angles ($0 \leq \theta_\mathrm{CR} \leq \pi/2$), with the angle depending on the emitted frequency (Yoshii et al., 1997; Muggli et al., 1999). Although in a warm plasma the M mode is still bounded from below by the lower cutoff frequency, neither the wavenumber nor the M mode frequency are bounded from above because the frequency $\omega_\mathrm{M}$ is always larger than $v_\mathrm{e}k$, with the M mode behaving as $\bar{\omega} \simeq v\bar{k}$ in the short wavelength limit ($k \to \infty$), i.e. like an electron 'sound' wave. This is shown in Fig. 1B by the lower bound of the orange area. Hence, the refractive index has an asymptotic upper bound of $n_\mathrm{M} = 1/v$. Consequently, radiation can no longer be emitted at all angles because the angle is bounded by $\cos\theta_\mathrm{CR} \simeq 1/n_\mathrm{M} = \bar{\omega}_\mathrm{M}/\bar{k} \geq v\bar{k}/\bar{k} = v$ (assuming $\beta_\mathrm{ph} \simeq 1$). Concurrently, the emitted frequency would go to infinity as the emission angle approaches $\theta_\mathrm{CR} = \arccos v$. Hence, the emission is limited to a 2D cone around the laser beam with angle $\arccos v$ in the **B**-perpendicular plane. For Earth's magnetosphere values, the emission cone is nearly all-encompassing with an angle of 89.9° whilst for a typical tokamak with electron thermal speed $v_\mathrm{e} = 5.5 \times 10^7$ m/s (Goedbloed et al., 2019), the angle narrows slightly to 79.4°. Even for a relativistically hot plasma that reached its maximal sound speed limit ($v^2 \simeq 0.3$), the cone angle is still as large as 56.8°. In general, the lowest frequency (defined by $n_\mathrm{M} = 1$) is emitted parallel to the laser beam whereas all higher frequencies ($n_\mathrm{M} > 1$) are emitted at an oblique angle between zero and $\arccos v$.

As discussed at length in De Jonghe and Keppens (2020), propagation perpendicular to the magnetic field differs significantly from oblique propagation due to the presence of avoided crossings at such angles. However, the M mode also behaves as $\omega \simeq v_{\mathrm{e}}k$ in the oblique short wavelength limit, such that $\theta_{\mathrm{CR}}$ is still limited by the electron thermal velocity $v_e$. Hence, Cherenkov radiation is limited to a 3D cone around the laser beam with angle $\arccos v$.

## Conclusion

It was shown that for a warm ion-electron plasma the angle at which laser-induced Cherenkov radiation is emitted (with respect to the laser beam) is limited to a cone by the electron thermal velocity (and thus temperature) of the plasma, for a laser beam propagating perpendicular to the applied magnetic field. This is not the case in a cold plasma description, where Cherenkov radiation can be emitted at all forward angles. Despite this difference, it should be noted that most radiation is emitted at or near the plasma frequency because the group speed is largest there, and consequently propagates (nearly) parallel to the laser beam, since $\theta_{\mathrm{CR}}(\omega_{\mathrm{p}}) = 0$.

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