Computation of neoclassical toroidal viscosity with the account of non-standard orbits in a tokamak

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Neoclassical toroidal viscous (NTV) torque [1] arises from the non-resonant part of 3D magnetic perturbations in tokamak plasmas. This includes external perturbations such as error fields, ripple and intentional 3D fields from extra coils to mitigate edge localized modes (ELMs) and internal perturbations from magnetohydrodynamic (MHD) modes [6]. Quantifying this torque is required for example to assess the risk of plasma rotation slowdown and subsequent resistive wall modes (RWMs). Another important application is neoclassical transport of (impurity) species densities, linked to the NTV torque via the flux-force relation [1, 2, 4]. One mechanism behind NTV torque is resonant diffusion due to orbital (not magnetic field) resonances. They occur when toroidal precession stops (superbanana resonance) or has a rational ratio to the bounce/transit frequency (drift-orbit resonance). If a 3D perturbation contains harmonics matching this ratio, a periodic excitation of the respective orbit occurs in the sense of wave-particle interaction. As in usual neoclassical theory, depending on collisionality and perturbation amplitude, three main transport regimes arise [1]: a non-linear regime in the collisionless limit, a resonant plateau regime, and collisional regimes. The drift-kinetic solver NEO-2 [4, 5] treats the latter two cases range for electrons, bulk ions and impurities, whereas the Hamiltonian code NEO-RT [2] focuses on the low-collisional limit and resonant plateau important for ion NTV in reactor conditions. NTV models usually rely on a local neoclassical ansatz treating deviations of orbits from flux surfaces as infinitesimal with the thin orbit limit for bounce and precession frequencies. This approach is limited in particular for ion NTV for two reasons: 1) orbit frequencies and even topology – e.g. potato orbits near the the magnetic axis – differ from infinitely thin passing and banana orbits on a flux surface. 2) Finite orbits radially re-distribute quantities such as torque density. The second point has been addressed earlier [3] while keeping thin orbit frequencies. Here we present the full orbit model for NEO-RT in the quasilinear limit.

The quasilinear evolution of the leading order distribution function f_0 due to Fourier harmonics $H_{\mathbf{m}}$ of a Hamiltonian in terms of canonical angles θ^k is

$$\frac{\partial f_0}{\partial t} = \sum_{\mathbf{m}} m_i \frac{\partial}{\partial J_i} Q_{\mathbf{m}}, \quad \text{where} \quad Q_{\mathbf{m}} = \frac{\pi}{2} |H_{\mathbf{m}}|^2 \delta(m_k \Omega^k - \omega) m_k \frac{\partial f_0}{\partial J_k}. \tag{1}$$

Here J_k and Ω^k are canonical actions and frequencies, respectively, and $Q_{\mathbf{m}}$ the driving term for resonant diffusion in action space. The unperturbed distribution is a quasi-Maxwellian,

$$f_0 = f_0(\psi^*, H_0) = \frac{n_0(\psi^*)}{(2\pi m T_0(\psi^*))^{-3/2}} \exp\left(\frac{e\Phi(\psi^*) - H_0}{T_0(\psi^*)}\right)$$

where $\psi^* = c p_\varphi / e$ and H_0 are normalized toroidal canonical momentum and unperturbed Hamiltonian, respectively. A static, $\omega = 0$, Hamiltonian perturbation driven by the non-axisymmetric corrugation of magnetic flux surfaces [2] is used here as a Fourier series expansion over θ^k,

$$H - H_0 = (2H_0 - 2e\Phi - \omega_{c0} J_\perp) \frac{\delta B}{B_0} = \mathrm{Re} \sum_{\mathbf{m}} H_{\mathbf{m}} \mathrm{e}^{i\mathbf{m}\cdot\theta}, \tag{2}$$

where $\delta B = B - B_0$ is the perturbation of magnetic field strength, $J_\perp$ is the canonical perpendicular momentum and $\omega_{c0} = eB_0/(mc)$. A conservation law for any moment of the distribution function integrated over the volume $V(r_0)$ limited by a flux surface $r(\mathbf{r}) < r_0$

$$A(t, r_0) = \int_{V(r_0)} \mathrm{d}^3 r \int \mathrm{d}^3 p \, f(t, \mathbf{r}, \mathbf{p}) a(\mathbf{r}, \mathbf{p}) \tag{3}$$

is obtained multiplying Eq. (1) with $a(\theta, \mathbf{J}) \Theta(r_0 - r(\theta, \mathbf{J}))$ where $\Theta(x)$ is a Heaviside step function and integrating over actions and angles,

$$\frac{\partial}{\partial t} A(t, r_0) + \Gamma_A(t, r_0) = S_A(t, r_0), \tag{4}$$

where the total flux is

$$\Gamma_A(t, r_0) = -\int \mathrm{d}^3\theta \int \mathrm{d}^3 J \, \delta(r_0 - r(\theta, \mathbf{J})) \sum_{\mathbf{m}} Q_{\mathbf{m}} a(\theta, \mathbf{J}) m_i \frac{\partial}{\partial J_i} r(\theta, \mathbf{J}), \tag{5}$$

and the integral source is

$$S_A(t, r_0) = -\int \mathrm{d}^3\theta \int \mathrm{d}^3 J \, \Theta(r_0 - r(\theta, \mathbf{J})) \sum_{\mathbf{m}} Q_{\mathbf{m}} m_i \frac{\partial}{\partial J_i} a(\theta, \mathbf{J}). \tag{6}$$

Setting $a = p_\varphi$ yields the integral source of the toroidal canonical momentum,

$$T_{\varphi\mathrm{box}}(r_0) = -2\pi^3 \sum_{\mathbf{m}} \sum_{\sigma=\pm 1} m_\varphi^2 \int \mathrm{d}H_0 \int \mathrm{d}J_\perp \int \mathrm{d}p_\varphi |H_{\mathbf{m}}|^2 \delta(m_k \Omega^k) \frac{\partial f_0}{\partial p_\varphi} \int_0^{\tau_b} \mathrm{d}\tau \, \Theta(r_0 - r(\mathbf{r}(\tau, \mathbf{J}))), \tag{7}$$

where m_φ is the toroidal harmonic index and time integration along the unperturbed orbit $\mathbf{r}(\tau,\mathbf{J})$ is over the full bounce period τ_b. Here, integration over the poloidal action and angle has been replaced with integration over total energy H_0 and orbit parameter τ and, therefore, can be performed using usual particle orbits in non-canonical phase space variables. To parameterize integration over p_φ we introduce the Poincaré cut $\nabla B \times \nabla \psi \cdot \nabla \varphi = 0$ which can be shown to cross all possible orbits in case of constant electrostatic potential within flux surfaces $\Phi = \Phi(\psi)$. This surface reduces to the $Z = 0$ plane in up-down symmetric configurations.

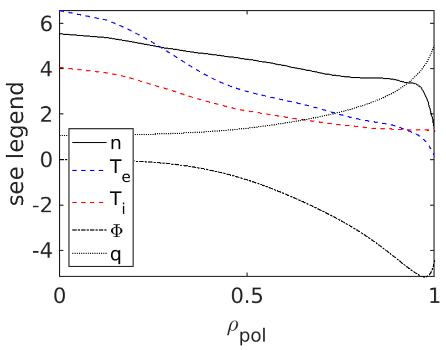

Figure 1: Fitted plasma profiles for ASDEX Upgrade discharge #32169: density n $[10^{19}\,\mathrm{m}^{-3}]$ (—), electron temperature T_e [keV] (– –), ion temperature T_i [keV] (– –), Electric potential Φ [kV] (–·), safety factor q (··).

The full orbit model in NEO-RT was applied to compute NTV torque in ASDEX Upgrade discharge #32169 (Fig. 1, [6]) with a helical core of toroidal mode number $n = 1$, based on a 3D MHD equilibrium modeled in VMEC. The radial electric field E_r is computed from measured toroidal rotation velocity using NEO-2. We compare two cases to the original thin orbit model of NEO-RT: One with temperature and E_r scaled down by a factor of 100, and one with actual profiles. The scaling is performed for two reasons: To reduce finite orbit width towards the thin orbit limit, and to remove a siginificant shift in the bounce frequency ω_b due to E_r. Results for integrated toroidal torque T_φ^{int} are shown in Fig. 2, and canonical frequencies at a specific flux surfaces are compared in Fig. 3. In the scaled case (Fig. 2a) the computed torque agree well between thin and full orbit models. This is to be expected looking at Fig. 3a, where canonical frequencies, and therefore orbital resonances nearly coincide. In contrast, toroidal torque from the full orbit model is significantly different for actual profiles (Fig. 2b). In contrast to the thin orbit limit, transport near $\rho_{\mathrm{pol}} = 0.4$ is absent for full orbits. Besides the larger orbit width, an explanation for this are modified canonical frequencies (Fig. 3b) and therefore resonances. Furthermore, near the magnetic axis, the torque from the collisionless full orbit model in NEO-RT matches a collisional computation in NEO-2 more closely than the collisionless thin orbit model. This is probably a coincidence, as transport is enhanced by non-standard orbits near the axis and collisions to a similar degree. Apart from that, the relatively close agreement of NEO-2 and thin orbit NEO-RT confirms major contributions from collisionless resonant transport. Up to $\rho_{\mathrm{pol}} = 0.6$ the full orbit model predicts substantially less ion torque than previous approaches.

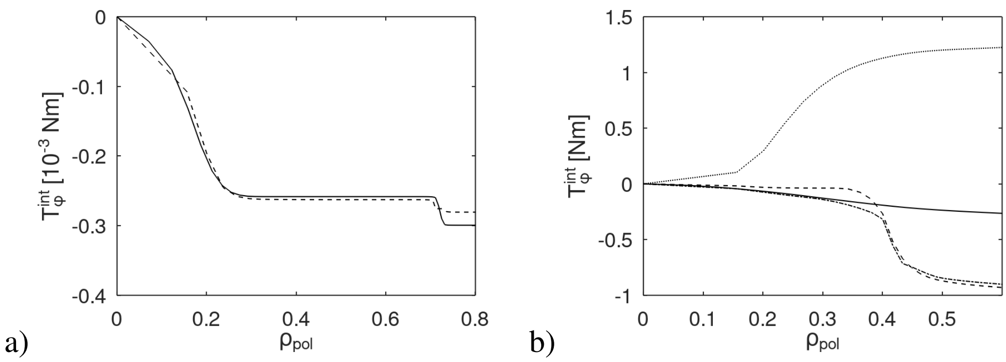

Figure 2: Integral torque $T_{\varphi}^{\mathrm{int}}$ over normalized poloidal radius $\rho_{\mathrm{pol}} \sim \sqrt{\psi_{\mathrm{pol}}}$. a) Deuterium ions in NEO-RT full ($-$) and thin ($--$) orbit model with temperature and electric field reduced by factor 100. b) Actual profiles, adding NEO-2 collisional model for ions ($-\cdot$) and electrons ($\cdot\cdot$).

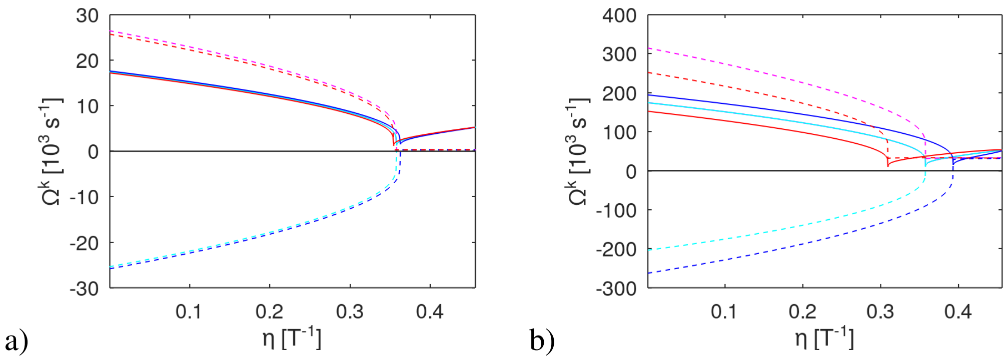

Figure 3: Canonical frequencies $\Omega^2 = \omega_b$ ($-$) and $\Omega^3 = \Omega_{tor}$ ($--$) at $\rho_{\mathrm{pol}} = 0.54$ from NEO-RT full and thin orbit model (light colors) with scaled field and temperature (a) and actual conditions (b). The radial electric field shifts trapped-passing boundary for opposite signs of $v_{\parallel}$.

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