Radial correlation reflectometry analysis in high-turbulence plasma scenario

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Introduction The radial correlation reflectometry (RCR) is a widely used technics that provides information on plasma turbulence characteristics. Probing plasma with multiple frequencies can simply determine the turbulence radial correlation length by the difference in the cut-off positions where the correlation of the signals disappears. It turned out that this approach is not always correct [1,2]. That stimulated theoretical investigation of the RCR and resulted in the non-linear theory of the RCR in 1D [3] and 2D [4] models. According to the developed models in the case of strong turbulence the signal spatial correlation length is a function of both the turbulence correlation length and its amplitude. Another method of the reflectometry signal analysis was developed for extracting information on the turbulence spectrum thus the turbulence amplitude [5]. This approach is based on the relation between the radial wave-number spectrum of the density fluctuations and the phase fluctuation wave-number spectrum of a reflectometer signal [6]. Theoretically the two methods can be combined for obtaining the information on both the turbulence amplitude and its radial correlation length under the conditions when the non-linear regime of the RCR takes place. This paper is devoted to the demonstration and verification of the possibility to use the two approaches of the RCR signal interpretation simultaneously. On the base of 2D simulation of a RCR experiment it is shown that this method allows us to resolve the turbulence amplitude and the turbulence radial correlation length.

Phase spectrum analysis Description of the phase variation spectrum analysis for O-mode probing in the RCR experiment is provided in this section. This method was described in details in [6] under the Born approximation. Since we are aimed at the RCR analysis in the strongly non-linear regime, the WKB approach will be used instead of the Born approximation. We consider here the slab plasma model. The Cartesian coordinates are chosen as follows: x is the direction of plasma inhomogeneity, a probing beam is launched along this axis; z axis corresponds to lines of external magnetic field; y axis is perpendicular to the x and z and stands for the poloidal coordinate. Also a linear density profile will be used in this model $n(x) = n_{max}\frac{x}{L}$. It will be convenient to express the density profile in terms of the critical density $n_c = n_c(\omega)$ and the cut-off position $x_c = x_c(\omega, L)$ for a given probing frequency ω, $n(x) = n_c\frac{x}{x_c}$. In the WKB approach the phase variation associated with the density turbulence $\delta n(x)$

$$\delta\phi(x_c) = -\int \frac{dq}{2\pi}\left[\sqrt{\pi}\frac{\omega}{c}\frac{x_c^{1/2}}{|q|^{1/2}}\frac{\delta n_q}{n_c}e^{-i\frac{\pi}{4}sign(q)}erf\left(\sqrt{iqx_c}\right)\right]e^{iqx_c} \tag{1}$$

where δn_q is the Fourier transform of the fluctuations $\delta n(x)$, q is the radial wave-number. It should be mentioned that the WKB description is applicable in the case of long wavelength turbulence. The error-function $erf(\sqrt{iqx_c})$ in (1) is almost a constant and equals 1 for the turbulence modes with $q > \frac{1}{x_c}$. Assuming the dominant contribution to the phase variation coming from the modes with $q > \frac{1}{x_c}$ we can neglect the error function in (1), then this equation becomes the natural definition for the phase perturbation spectrum. The relation between the turbulence spectrum $S_{\delta n}(q)$ and the phase spectrum $S_{\delta\phi}(q)$

$$S_{\delta\phi}(q) = \pi\frac{\omega^2}{c^2}\frac{x_c}{n_c^2}\frac{1}{|q|}S_{\delta n}(q) \tag{2}$$

In the RCR experiment we have a set of N cut-offs from minimal to maximal values $x_{cmin},\ldots,x_{cmax}$, so the phase spectrum can be evaluated. Then the average (within the radial window $\{x_{cmin},x_{cmax}\}$) turbulence amplitude is known according to the discrete Parseval's theorem

$$< \delta n^{rms} >_{\{x_{cmin},x_{cmax}\}} = \frac{1}{N}\sqrt{\sum_{q=q_{min}}^{q_{max}} S_{\delta n}(q)} \tag{3}$$

where $q_{min} = \frac{2\pi}{x_{cmax}-x_{cmin}}$ is determined as the minimal wave number resolved by the discrete Fourier transform and $q_{max} = 2k(\omega_{max},x_{cmin})$ is the maximal wave number which fulfills the back scattering Bragg's rule in the considered radial window, k is the probing wave number with the maximal frequency ω_{max} evaluated in the position x_{cmin}.

CCF analysis Analysis of the cross-correlation function in the strongly non-linear regime is described in [3,4] for both 1D and 2D plasma models correspondingly. One of the main results in those works is the explicit expression for the signal cross-correlation function

$$CCF(\Delta x_c, \Delta t = 0) = e^{-\frac{\Delta x_c^2}{l_{ceff}^2}} \tag{4}$$

with the correlation length

$$l_{ceff} = \frac{c}{\omega}\frac{n_c}{\delta n^{rms}}\sqrt{\frac{\sqrt{\pi}l_c}{L_{loc}}} \tag{5}$$

where $L_{loc} = \frac{1}{n(x)} \frac{dn}{dx}\Big|_{x=x_c}$ is the local plasma gradient, the turbulence radial correlation length is defined as $l_c = \frac{1}{\sqrt{\pi}} \int d\Delta x CCF_{\delta n}(\Delta x)$, $CCF_{\delta n}(\Delta x)$ is the turbulence cross-correlation function in 1D model. If one considers 2D geometry $\delta n = \delta n(x,y)$ with the slab medium approximation $n = n(x)$ the expression (4) remains the same, the difference is only in the turbulence cross-correlation function definition $CCF_{\delta n}(\Delta x) = CCF_{\delta n}(\Delta x, \Delta y = 0)$. The expression (4) is derived in the regime with the strong phase variation $< \delta\phi^2 > \gg 1$,

$$< \delta\phi^2 > \approx \sqrt{\pi} \frac{\omega^2}{c^2} x_c l_c \frac{(\delta n^{rms})^2}{n_c^2} Ln\left(\frac{x_c}{\sqrt{\pi} l_c}\right) \tag{6}$$

another criterion of the CCF description is that the turbulence amplitude is not too big $\frac{\delta n^{rms}}{n_c} \ll \frac{l_c}{x_c}$ so only one cut-off exists.

Numerical simulation of the RCR In order to verify the possibility of extraction the information about the both turbulence amplitude and its correlation length in a RCR experiment simultaneously by means of the two described methods numerical simulations of the RCR experiment was fulfilled. The simulations are performed in the 2D geometry by the full-wave code IPF-FD3D. The linear density profile (in the slab model) was used in the simulation with $L = 16.2$cm, the maximal density $n_{max} = 1 \times 10^{14}\text{cm}^{-3}$. The probing frequency range $f \in [50, 80] GHz$ with the frequency step $\Delta f = 100$MHz. This frequency range corresponds to the set of cut-offs from $x_{cmin} = 4.9$cm to $x_{cmax} = 12.4$cm. The probing beam width is $\rho = 2$cm. The turbulence is homogeneous, its amplitude is defined as $A = \frac{\delta n^{rms}}{n_c(80GHz)}$. The turbulence correlation length corresponds to the definition $l_c = \frac{1}{\sqrt{\pi}} \int d\Delta x CCF_{\delta n}(\Delta x)$. The simulations are performed for different turbulence amplitudes A and different turbulence spectra $S_{\delta n}(q)$, the results (the phase spectrum and CCF) are averaged over 1000 random turbulence realizations.

A symmetric Gaussian spectrum was used for the first set of simulations $S_{\delta n}(q) \propto \exp\left(-\frac{q_\perp^2}{4 l_{cg}^2}\right)$ with the correlation length $l_{cg} = 1cm$. The results of the phase spectrum analysis (the amplitude A_{num}) and the CCF analysis (l_{cnum} according to (5) and taking into account measured l_{ceff} and known A_{num}) are demonstrated in the table 1. Relation of the turbulence correlation length to the cut-off position in this case $\frac{l_{cg}}{x_{cmax}} \approx 0.08$, this parameter specifies for which turbulence amplitudes A the theory is applicable. The analogous simulations were performed in the same model and for the same turbulence amplitude range, but for a realistic turbulence spectrum. This spectrum is adopted from the experimental measurements on ASDEX Upgrade [7]. This spectrum provides the turbulence correlation length $l_{cr} \approx 0.75$cm. Then the relation $\frac{l_{cr}}{x_{cmax}} \approx 0.06$. The results of the simulations with the realistic turbulence spectrum are shown in the table 2. One can conclude that the phase spectrum analysis provides pretty correct values for the

№	A	A_{num}	$< \delta\phi^2 >$	l_{cnum}, cm
1	0.013	0.010	1.9	0.48
2	0.025	0.023	7.6	0.50
3	0.050	0.045	30.4	0.86
4	0.063	0.058	47.5	1.20
5	0.076	0.070	68.5	1.32
6	0.101	0.094	121.7	1.81
7	0.126	0.120	190.2	2.29

Table 1: Simulations with the Gaussian turbulence spectrum. Used in the simulation turbulence amplitude A, the measured amplitude A_{num}, the measured correlation length l_{cnum}, the averaged squared phase variation $< \delta\phi^2 >$

№	A	A_{num}	$< \delta\phi^2 >$	l_{cnum}, cm
1	0.013	0.009	1.6	0.42
2	0.025	0.024	6.5	0.46
3	0.050	0.051	26.2	0.77
4	0.063	0.066	40.9	0.93
5	0.076	0.080	59.0	1.17
6	0.101	0.109	104.8	1.66
7	0.126	0.135	163.7	2.31

Table 2: Simulations with the realistic turbulence spectrum. Used in the simulation turbulence amplitude A, the measured amplitude A_{num}, the measured correlation length l_{cnum}, the averaged squared phase variation $< \delta\phi^2 >$

turbulence amplitudes A_{num} for all the tested conditions. Taking into account the applicability criteria for the CCF analysis the simulations number 3,4,5 for the Gaussian spectrum (table 1) and 3,4 for the experimental spectrum (table 2) should be in agreement with the theory.

Conclusion The possibility of measuring the turbulence amplitude and the turbulence correlation length simultaneously in the high-turbulence plasma scenario in a RCR experiment is demonstrated. The phase spectrum analysis provides pretty accurate values of the turbulence amplitude (the relative error is about 10% or smaller) for all the simulated conditions with the different turbulence amplitudes and spectra. It is shown that the turbulence correlation length is in quite good agreement with the prediction of the theory under the conditions when the non-linear RCR theory is applicable. In order to extend applicability of the proposed RCR analysis method a correction coefficient (depending on A) to l_c can be introduced, but this requires analysis of more simulations and it is planned for the further work.

Acknowledgements The analytical treatment is performed under the Ioffe Institute state contract 0040-2019-0023 whereas the numerical modelling was performed under the Ioffe Institute state contract 0034-2021-0003.

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