Hydrodynamic modeling of self-generated magnetic fields by ALE methods

M. Kucharik[1], J. Nikl[1,2,3], J. Limpouch[1]

[1] *Faculty of Nuclear Sciences and Physical Engineering, Czech Technical University in Prague, Prague, Czech Republic*

[2] *ELI-Beamlines, Institute of Physics, Czech Academy of Sciences, Prague, Czech Republic*

[3] *Institute of Plasma Physics, Czech Academy of Sciences, Prague, Czech Republic*

Abstract

Arbitrary Lagrangian-Eulerian (ALE) methods belong to the most popular approaches for hydrodynamic simulations of laser/target interactions [1]. They benefit from improved accuracy of the simulation due to the Lagrangian motion of the computational mesh, as well as robustness resulting from a regular mesh optimization followed by conservative interpolation (remap) of all quantities between the meshes. In recent years, we were interested in modeling of spontaneous magnetic field (SMF) generation [2, 3] resulting from crossed gradients of electron temperature and density, known as the Biermann battery effect. Such models have been implemented in our developed 2D cylindrical Prague ALE (PALE) code [1], containing also all necessary physical models (EOS, laser absorption, thermal conductivity, etc.). Here, we present performance of the second-order accurate extension of the SMF generation model [4] in the ALE framework on selected realistic tests.

Introduction

Hydrodynamic modeling is a useful tool, helping to interpret processes happening during laser/target interactions in ICF-related applications. In general, it can be performed in one of two reference frames. In the Eulerian frame, a static computational mesh is employed, while the mesh typically moves with the fluid in the Lagrangian frame. The Lagrangian framework is usually preferred in laser/target simulations as the computational mesh automatically suits strong compressions and expansions typically present in this type of problems, and keeps high resolution at the shock waves. To eliminate potential mesh degeneracies and increase the robustness of the simulations, the arbitrary Lagrangian-Eulerian (ALE) methods [5] has been developed, rezoning (untangling and smoothing) the mesh continuously and remapping (interpolating) all fluid quantities conservatively to the rezoned mesh.

In our group, the Prague ALE (PALE) code [1] has been developed. This code incorporates the ALE algorithm in 2D cylindrical $r-z$ geometry together with all necessary physics models, in particular laser beam absorption model, thermal conductivity model, two-temperature model, several realistic equations of state, phase transition model, etc. It has been demonstrated [6, 7, 8] that this code realistically predicts processes for a wide variety of laser/target parameters. A simple model of spontaneous magnetic field (SMF) generation via the Biermann battery effect has been added recently [2] and generalized to the second order of accuracy for non-rectangular meshes [4]. This model can estimate the generated SMF magnitude and location at the high-gradient regions in the PALE code. In this paper, we demonstrate the application of the developed model for a realistic laser/target problem. Alternatively, MHD simulation of laser-target interaction may be carried out in an advanced curvilinear finite element framework [9, 10].

Hydrodynamic Model

A compatible staggered Lagrangian solver [11] is used to solve the Euler equations in the 2D cylindrical Lagrangian framework, followed by the Winslow mesh smoothing method [12] and sub-zonal remap [13] with a posteriori mass redistribution (repair) [14] guaranteeing local bound preservation. The energy equation contains two additional terms – a parabolic term representing thermal conductivity heat exchange, approximated by the support operators method [15, 16], and a source term introducing absorbed laser beam energy, modeled by the stationary solution of Maxwell equations [17]. The QEOS equation of state [18] consistently interpolated by the HerEOS library [19] is used.

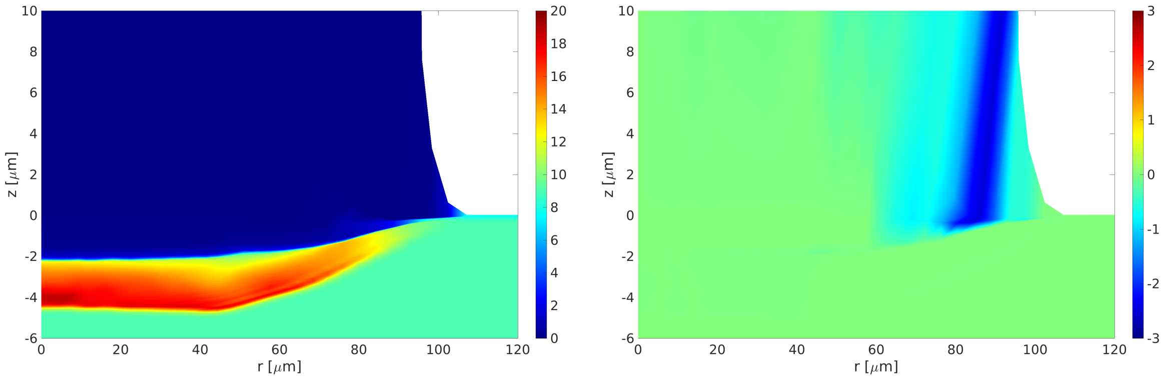
The magnetic field $\vec{B}$ generation in the Gaussian units is described by the Biermann battery term in the form

$$\frac{\mathrm{d}\vec{B}}{\mathrm{d}t} = -\nabla \times \vec{E} = -\frac{c}{e}\nabla\left(\frac{1}{n_e}\right) \times \nabla p_e \,,$$

where n_e is electron density, p_e electron pressure, c stands for the speed of light and e for the electron charge. In 2D axisymmetric $r-z$ geometry, only the angular field component $\vec{B} = (0,0,B_\varphi)$ is considered [2], with electron pressure and harmonic electron density derivatives resulting from a parabolic reconstruction guaranteeing second order of convergence for general meshes [4]. Remap of magnetic field is done in the form of magnetic energy density $b_\varphi = \frac{1}{\mu_0} B_\varphi^2$.

Numerical Example

In [2], we have demonstrated convergence of the model in the case of an analytical magnetic field generation test, and its ability to approximate the generated SMF distribution with reasonable magnitude for realistic laser/target setup. The model was improved to the second order of accuracy for non-rectangular meshes in [4], which is crucial in the case of its application in the Lagrangian deformed meshes. Here, we demonstrate the application of the model for data corresponding to a real experiment [20, 21] performed at the PALS laser facility.



(a) ρ [g/cm^3] (b) B_φ [MGs]

Figure 1: *Plasma density* ρ [g/cm^3] *and generated SMF* B_φ [MGs] *profiles for Cu target at triangular pulse maximum* $t = 250$ ps.

The massive Cu target at room temperature is irradiated by a laser pulse with the maximum intensity $I_{\max} = 10^{16}\,\mathrm{W/cm^2}$, containing 500 J of energy. The laser beam with the wavelength of 1.315 μm is super-Gaussian in space with FWHM 50 μm, and has a symmetric triangular shape in time, reaching the maximum at $t = 250$ ps. A flat shock wave is generated, moving inside the target, while the evaporated low-density plasma moves in the opposite direction, as can be seen in Figure 1 (a). The generated magnetic field develops mainly in the evaporated material at the critical density, close to the edge of laser beam, where the density and temperature gradients are largest. In Figure 3 of [21], it is shown that a similar simulation employing a Biermann term model leads to a maximum magnitude of about 2.5 MGs at time of pulse maximum, which is located close to $r \approx 100\,\mu$m. This corresponds well to the profile presented in Figure 1 (b). A different shape of the profile along the z direction can be observed, resulting mainly from the Lagrangian nature of the simulation, and consequently long computational cells due to high velocity of expanding plasma.

On the other hand, in experiments [20] and more advanced simulations [21], differences mainly in the SMF distribution can be observed, resulting from two main reasons. First, in our simple model, we ignore the backward influence of the generated SMF on the plasma. And second, the model of paper [21] includes additionally magnetic diffusion and anomalous resistivity for hot electrons due to the ion acoustic instability. Our model is therefore satisfactory for the estimates of the maximum generated SMF magnitude and its location, but more detailed study of the SMF distribution requires a full MHD description with richer physics model, see [3, 10] for an example.

Conclusions

The development of a magnetic field model and its integration to the staggered hydrodynamic ALE code has been described, and its application on a typical laser/target setup has been tested. We have shown that it provides results comparable to other simulation codes based on similar models and allows to estimate magnetic field magnitude and location approximately. On the other hand, for detailed magnetic field distribution studies, a full MHD description with additional physics models provides more accurate results [10].

Acknowledgments

This work was supported in parts by the European Regional Development Fund project CZ.02.1.01/0.0/0.0/16_019/0000778, the Czech Science Foundation project 19-24619S, the Czech Technical University grant SGS22/184/OHK4/3T/14, the EUROfusion project CfP-FSD-AWP21-ENR-01, and the Czech Ministry of Education project RVO 68407700.

References

[1] R. Liska et al. In *Finite Volumes for Complex Applications VI Problems & Perspectives*, volume 4 of *Springer Proceedings in Mathematics*, pages 857–873. Springer, 2011.

[2] M. Kucharik et al. In *Europhysics conference abstracts, 47th EPS Conference on Plasma Physics*, volume 45A, page P1.2020, 2021.

[3] J. Nikl et al. In *Europhysics conference abstracts, 47th EPS Conference on Plasma Physics*, volume 45A, page P1.2022, 2021.

[4] M. Kucharik et al. *AIP Conference Proceedings*, 2021. Accepted.

[5] C. W. Hirt et al. *Journal of Computational Physics*, 14(3):227–253, 1974.

[6] A. Picciotto et al. *Physical Review X*, 4(3):031030, 2014.

[7] J. Badziak et al. *Laser and Particle Beams*, 35:619–630, 2017.

[8] L. Giuffrida et al. *Physical Review E*, 101(1):013204, 2020.

[9] J. Nikl et al. In *WCCM-ECCOMAS2020, Scipedia*, 2021.

[10] J. Nikl et al. *Journal of Computational Physics*, 464:111158, 2022.

[11] E. J. Caramana et al. *Journal of Computational Physics*, 146(1):227–262, 1998.

[12] A. M. Winslow. Technical Report UCRL-7312, Lawrence Livermore National Laboratory, 1963.

[13] R. Loubere and M. Shashkov. *Journal of Computational Physics*, 209(1):105–138, 2005.

[14] M. Kucharik et al. *Journal of Computational Physics*, 188(2):462–471, 2003.

[15] M. Shashkov and S. Steinberg. *Journal of Computational Physics*, 129(2):383–405, 1996.

[16] R. Liska and M. Kucharik. In *Proceedings of EQUADIFF 11, International Conference on Differential Equations*, pages 213–222. STU, 2007. ISBN 978-80-227-2624-5.

[17] J. Nikl et al. *Advances in Computational Mathematics*, 45:1953–1976, 2019.

[18] R. M. More et al. *Physics of Fluids*, 31(10):3059–3078, 1988.

[19] M. Zeman et al. *Computers & Mathematics with Applications*, 78(2):483–503, 2019.

[20] T. Pisarczyk et al. *Physics of Plasmas*, 24(10):102711, 2017.

[21] K. Jach et al. *Physics of Plasmas*, 28(9):092704, 2021.