Dynamic error fields derivation by inverting a validated interpretative perturbations model

I.G. Miron and JET Contributors*

National Institute for Lasers, Plasma and Radiation Physics, Magurele-Bucharest, Romania

Model description

A dynamic model of the perturbations [1] have been used in order to further derive the corresponding magnetic error field modes by means of the perturbed system of equations providing the dynamics of the neoclassical resistive perturbations shown in (79) of Ref. [1]:

$$\sum_{j,k}\sum_{i=0}^{3}\left(F_{mni}^{jk}\frac{\partial^i\Phi_s^{jk}}{\partial t^i}+G_{mni}^{jk}\frac{\partial^i\Phi_s'^{jk}}{\partial t^i}\right)=-E^{mn}(t) \tag{1}$$

The system of equations provides the outer plasma (including various external resistive structures) perturbed equations localized at the magnetic surface of radial coordinate r_s where the perturbation is developing. F_{mni}^{jk} and G_{mni}^{jk} are exactly derived coefficients showing the dependence of the outer plasma resistive wall and passive coils system. The active part of the latter is comprised by the E^{mn} quantity. The local perturbation is searched for as the perturbed flux Φ_s^{jk} where $-\partial\Phi_s^{jk}/\partial t$ is the Fourier term of the perturbed electric scalar potential at r_s. $\Phi_s'^{jk}$ is the radial derivative of the perturbed flux. If we parametrize the error field in the absence of the plasma in the same manner as the perturbation by means of a vacuum flux Ψ_{EF}, equation (72) from Ref. [1] showing the response of the plasma in the presence of the error field [2] becomes

$$\Delta_s'\Psi_s^{mn}+2m\Psi_{EF}^{mn}=\alpha_w^{mn}\frac{\partial\Psi_s^{mn}}{\partial t}+\alpha_{BS}^{mn}\Psi_s^{mn} \tag{2}$$

α_w^{mn} and α_{BS}^{mn} are (73) and (74) from Ref. [1] corresponding to the island dynamics and the bootstrap term effects. The $\Psi^{mn}\cong(\mathrm{i}/R_0)(m/q-n)\Phi^{mn}$ parametrization has been used. Therefore by introducing the error field the perturbed system of equations (1) now is

$$\sum_{j,k}\sum_{i=0}^{2}J_{mni}^{jk}\frac{\partial^i\Phi_{EF}^{jk}}{\partial t^i}=\sum_{j,k}\sum_{i=0}^{3}K_{mni}^{jk}\frac{\partial^i\Phi_s^{jk}}{\partial t^i}+E^{mn}(t) \tag{3}$$

J_{mni}^{jk} and K_{mni}^{jk} are derived by using the jump of the derivative of the logarithm of the flux perturbation across the corresponding inertial-resistive layer, for the error field and for the perturbation

*See the author list of “Overview of JET results for optimising ITER operation” Joelle Mailloux et al. 2022 Nucl. Fusion mode, respectively. The right hand part of (3) uses our validated direct model[1] results for the involved perturbations through our derived quasi-analytic expression that it has been previously proven to match the experimental results by means of its amplitude and the frequency. Only then we are entitled to further derive the involved error field mode quantities. q_s and s_s are the safety factor and the shear at r_s. We are searching for the solution Φ_{EF}^{jk} as the sum of the general solution of the homogenous part of the system (3) and a particular solution of the inhomogeneous one. By Laplace transforming (3), applying the long division of the fraction and finally using the partial fraction decomposition method, we get the particular solution as

$$\begin{aligned} \Lambda_p^l(t) &= \frac{1}{c_{2L}} \sum_{\lambda=0}^{2L} \sum_{i=1}^{2L} \left(K_\lambda^l + \frac{A_{\lambda s}^l}{s_i} + \frac{B_{\lambda s}^l}{s_i + \mathrm{i}\Omega_{MP}} + \sum_{p=1}^{7L} \frac{C_{\lambda ps}^l}{s_i - \tau_p} \right) \left[s_i^\lambda \Big/ \prod_{\substack{j=1 \\ j \neq i}}^{2L} (s_i - s_j) \right] \exp(s_i t) \\ &+ \frac{1}{c_{2L}} \left[\frac{A_{\lambda s}^l \delta_{\lambda 0}}{\prod_{i=1}^{2L} s_i} + \frac{B_{\lambda s}^l (-\mathrm{i}\Omega_{MP})^\lambda}{\prod_{i=1}^{2L} (s_i + \mathrm{i}\Omega_{MP})} \exp(-\mathrm{i}\Omega_{MP} t) + \sum_{p=1}^{7L} \frac{C_{\lambda ps}^l \tau_p^\lambda}{\prod_{i=1}^{2L} (s_i - \tau_p)} \exp(\tau_p t) \right] \end{aligned} \tag{4}$$

All the coefficients are exactly derived expressions of the dynamic plasma quantities provided by the diagnostics data tables and of the outer plasma resistive structures parameters. $\{s_i\}_{i=1,..,2L}$ are the $2L$ roots of the $D(\tau) = 0$ equation where

$$D = \sum_{\lambda=0}^{2L} \tau^\lambda \sum_{\substack{l_1,..,l_L=1 \\ distinct}}^{L} sgn(l_1,..,l_L) \sum_{\substack{\alpha_1,...,\alpha_L=0 \\ \alpha_1+...+\alpha_L=\lambda}}^{2} \prod_{\nu=1}^{L} J_{\nu\alpha_\nu}^{l_\nu} \tag{5}$$

is the Leibniz description of the characteristic determinant of the Laplace transformed system of error field equations (3). $L = (m_2 - m_1 + 1)(n_2 - n_1 + 1)$ and $l = m - m_1 + 1 + (n - n_2)(m_2 - m_1 + 1)$ where m and n span $m_1 < m < m_2$ and $n_1 < n < n_2$. $\{\tau_p\}_{p=1,..,7L}$ are the roots of the $\Delta(\tau) = 0$ equation from (96) of Ref. [1]. The general error field solution is finally

$$\Phi_{EF}^l = \sum_{i=1}^{2L} c_i v_i^l \exp(\lambda_i t) + \Lambda_p^l + \alpha^l + \beta^l \exp(-\mathrm{i}\Omega_{MP} t) \tag{6}$$

λ_i and v_i are the eigenvalues and the eigenfunctions of the $2L \times 2L$ matrix

$$\mathbf{M} = \begin{pmatrix} \mathbf{O}_L & \mathbf{I}_L \\ -(\mathbf{J}_2^T)^{-1} \cdot \mathbf{J}_0^T & -(\mathbf{J}_2^T)^{-1} \cdot \mathbf{J}_1^T \end{pmatrix} \tag{7}$$

and c_i are constants left to be derived from the initial conditions. A suitable assumption is to consider the only presence of the error field at $t = 0$. α^l and β^l are exactly derived solution related to the plasma external resistive wall and coils system. Recall the one-to-one relations between l and the mode numbers (m,n) i.e. $m = l + m_1 - 1 + (m_2 - m_1 + 1)[(l-1)/(m_2 - m_1 + 1)]$ and $n = n_1 + [(l-1)/(m_2 - m_1 + 1)]$[3]. The square bracket here means the integer part.

Model testing against JET results

The solution (6) has been tested during various discharges at JET. For instance for the JET shot no. 77635 figures 1(a) and 1(b) shows the good match between our calculated perturbation amplitude and frequency respectively, using our direct model derived perturbed flux function Ψ_s^{mn}[1]. Based on this good matching result we are encouraged to derive trustworthy values for the error field flux amplitude by using relation (6) that is shown in figure 1(c). Instead of directly considering the measured perturbed magnetic flux associated to the perturbation, we use our calculated one that provides a quasi-analytic solution for the corresponding error field mode if our calculated perturbation matches the measured quantity. According to the observations there is an early $2/1$ mode that locks from 3.65s until 3.8s. Although no error field reference has been assumed, our calculations confirm that the calculated corresponding error field mode is pretty high (approx. 40% of the perturbation amplitude) in order to trigger the mode locking effect. The measured data that is compared against our calculated results in figures 1(a) and 1(b) is provided by the JET Mode Analysis MHD python code [4] concerning the modes amplitude, frequency and radial location. The rest of the plasma quantities we use in our calculations is provided by the diagnostic data tables at JET. On the other hand figure 2 shows a similar behavior for two JET shots having no mode locking effect reported. Our derived error field amplitude calculations following the good match between the measured and the calculated perturbations amplitude seems to justify the lack of the mode locking effect: the error filed amplitude, in both cases, is pretty low compared to the perturbation one. No mode frequency comparison is shown for the simple reason that the toroidal plasma rotation collected from using the charge exchange recombination spectroscopy diagnostics is not provided. It has been used the perturbations spectrogram to provide the local toroidal plasma rotation instead.

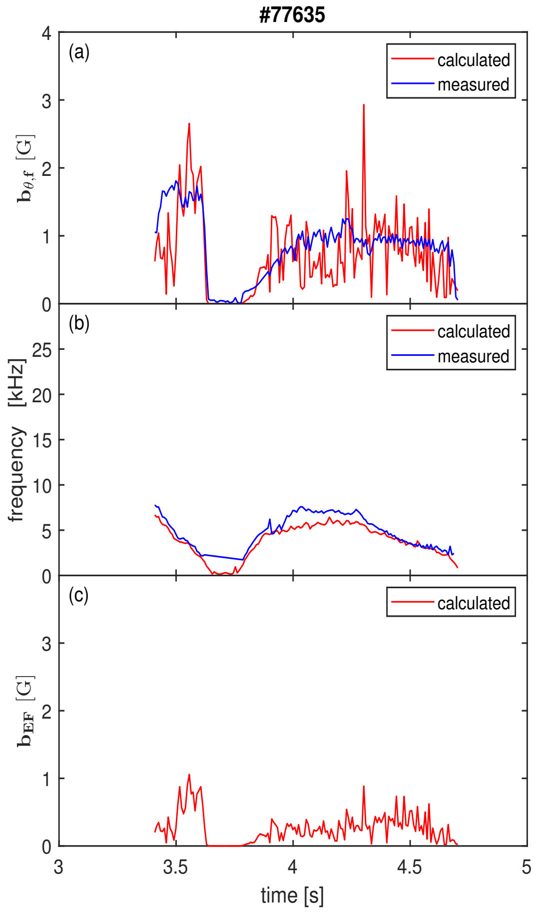

Figure 1: *JET 77635 shot:* $2/1$ *calculated vs measured perturbation (a) amplitude, (b) frequency and (c)* $2/1$ *calculated error field amplitude*

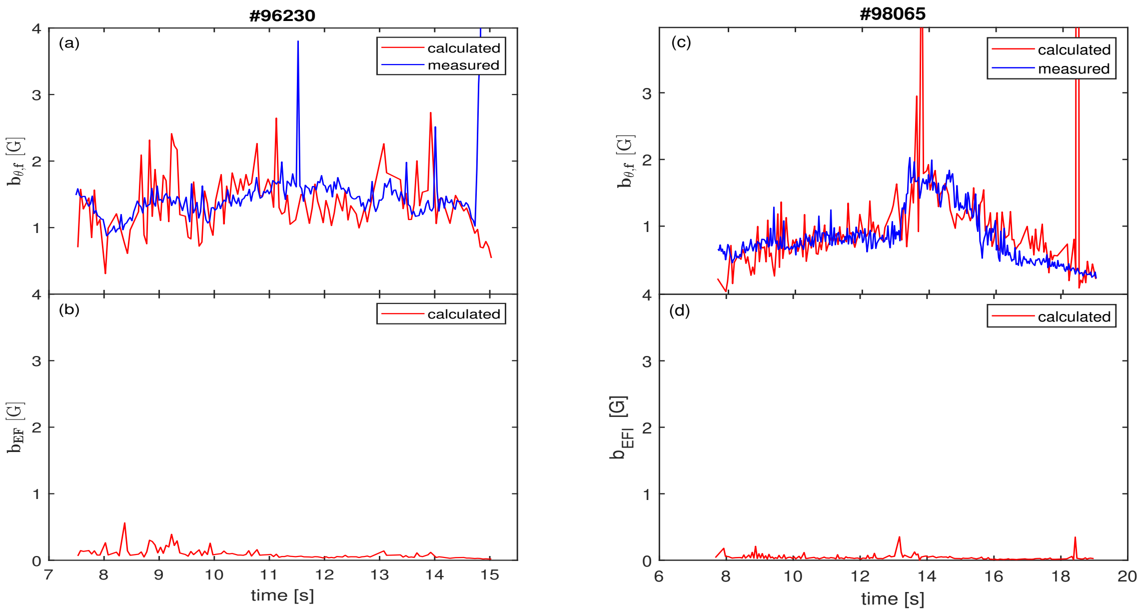

Figure 2: *JET 96230 and 98065 shots measured vs calculated amplitude of the perturbations ((a),(c)) and of the corresponding error fields ((b),(d)), respectively.*

Keeping into account that no mode locking occurs in order to alter the local plasma rotation, we think that this approach is reasonable. To conclude a dynamic expression for the amplitude of the intrinsic magnetic error field has been provided by inverting an interpretative model that offers good matching calculated versus measured results at JET. By relying on the mentioned direct model testing and validation we believe that our approach in delivering our derived error field results is also valid and reliable.

Acknowledgements

This work has been carried out within the framework of the EUROfusion Consortium, funded by the European Union via the Euratom Research and Training Programme (Grant Agreement No 101052200 — EUROfusion). Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union or the European Commission. Neither the European Union nor the European Commission can be held responsible for them.

References

[1] I.G. Miron, Nuclear Fusion **61**, 106016 (2021)

[2] R. Fitzpatrick, Physics of Plasmas **21**, 092513 (2014)

[3] I.G. Miron, Plasma Phys. Controlled Fusion **50**, 095003 (2008)

[4] E. Giovannozzi, (https://users.euro-fusion.org/openwiki/index.php/MHD-python_code) (2020)