PIC Simulations of the Interaction between Self-Modulation in the Front and Rear of a relativistic Proton Bunch in Plasma

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A long relativistic proton bunch propagating in plasma can undergo self-modulation (SM) [1], which transforms it into a microbunch train that resonantly drives wakefields [2]. These wakefields can be used to accelerate particle bunches to high energies [3]. To produce accelerated bunches with reproducible properties, the SM must be seeded [4], so that its phase and amplitude can be repeated event by event. When there is more than one seed, two SM processes could develop simultaneously and interact. This is particularly interesting for the future of the Advanced Wakefield Experiment (AWAKE) [5].

In the linear regime, the growth of the amplitude of the transverse wakefields is given by $W_{\perp}(\xi,z) = W_{\perp,0}\exp(\Gamma(\xi,z)z)$, where ξ is the position along the bunch, z the propagation distance in plasma, $W_{\perp,0}$ is the initial amplitude of the wakefields, and Γ is the growth rate of the modulation which depends on the proton bunch and plasma densities [8, 9]. When using two seeds, each self-modulating part has a different $W_{\perp,0}$, which, together with the ξ-dependence of the proton bunch density, leads to a different growth in each part.

Table 1: Main simulation parameters, rms is root mean square.

Plasma	Value
Plasma density	$2\times10^{14}\,\mathrm{cm}^{-3}$
Plasma radius	0.1 cm
Proton bunch	
rms radius (σ_{r0})	200 µm
rms length (σ_z)	7.5 cm
Norm. emit. (ε_N)	3.6 mm mrad
Den. cut pos. (ahead of center)	15 cm
Energy	400 GeV
Population (protons)	3×10^{11}
Electron bunch	
rms radius (σ_{r0})	270 µm
rms length (σ_z)	300 µm
Norm. emit. (ε_N)	3 mm mrad
Energy	18.89 MeV
Population (electrons)	3.43×10^{9}

We present here a numerical study using the quasi-static particle-in-cell code LCODE [7] with parameters similar to those in experiments. We use 2D axisymmetric geometry. In order to study the effect that the self-modulating bunch front has on the bunch rear in a way that could also be reproduced in experiments, we use the two seeding mechanisms available in experiments: ionization front seeding [4] (replaced by a density cut in simulations) and electron bunch seeding [6]. We place the electron bunch inside of the proton bunch, in contrast with the usual electron bunch seeding in which it is ahead of the proton bunch ($2.5\,\sigma_z$ in the experiment) [6]. For this study, we consider that the plasma density is a step function at the beginning of the plasma, i.e. there is no plasma density ramp. The main simulation parameters are in Table 1.

The seeding by one mechanism or the other is influenced by the position of the electron bunch and density cut, and the densities of the electron and proton bunches, and of the plasma. For

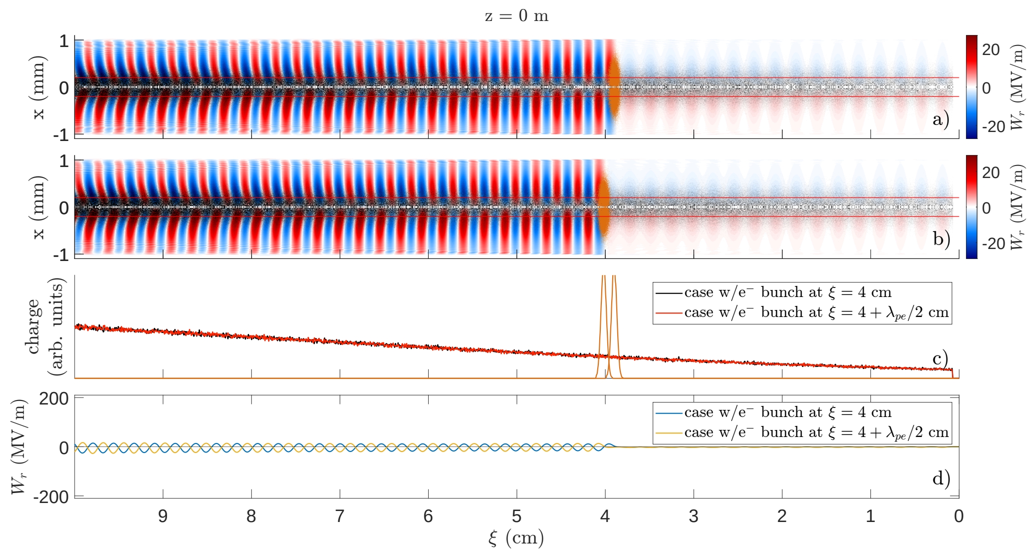

Figure 1: [(a) and (b)] Transverse wakefields (blue and red), proton bunch density (black) and electron bunch density (orange). (c) Charge profiles of the proton (red and black) and electron (orange) bunches integrated up to $x = \sigma_{r0} = 0.2$ mm [red lines in (a)]. (d) Lineouts of the transverse wakefields at $x = \sigma_{r0}$.

example, an electron bunch with a high enough charge placed close to the density cut can seed the SM, i.e. the position of the microbunches in the train at the end of the plasma is determined by the initial position of the electron bunch. When we keep repeating the simulation reducing the electron bunch charge each time, there is a charge below which the density cut does the seeding (in an extreme case, when the charge is reduced to 0 pC).

In the transition between being seeded by the electron bunch or by the density cut, changing the position of the electron bunch has interesting effects on the microbunch train, which can be used to study the physics of SM. Figures 1 (a) and (b) show the initial transverse fields driven by the electron bunches at the beginning of the plasma, which have an amplitude of ≈ 20 MV/m, larger than the ones driven by the density cut. The wakefields are focusing and defocusing, a difference with the seed wakefields from the density cut which are only focusing for the protons. The seed electron bunches are placed at $\xi = 4$ and $\xi = 4 + \frac{\lambda_{pe}}{2}$ cm behind the density cut (Fig. 1 [a-c]), where λ_{pe} is the plasma oscillation wavelength. The wakefields they drive are therefore shifted by $\lambda_{pe}/2$ with respect to each other, as indicated by the lineouts in Fig. 1 (d). The adiabatic response of the plasma to the proton bunch leads to a focusing field for the protons all along the bunch, making the fields driven by the seed electron bunch asymmetric around 0 MV/m.

Figure 2 shows that, with the parameters chosen here, the growth of the transverse fields is larger behind the electron bunch than behind the density cut, up to $z \approx 3.5$ m, at which point microbunches are already formed in the back of the bunch. This is due to the overall higher initial wakefields and higher proton density in the region behind the electron bunch (Fig. 1 [c-

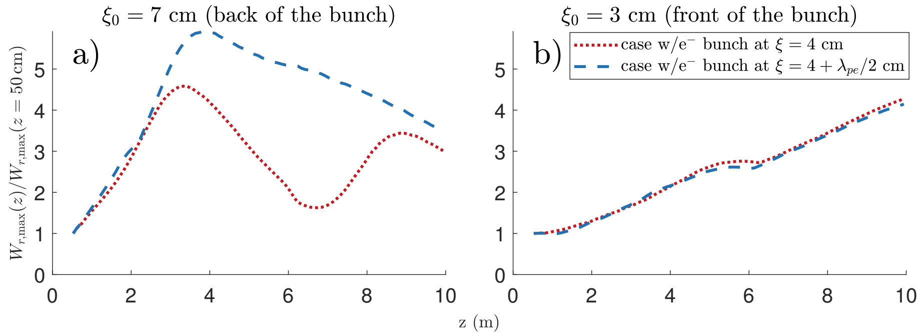

Figure 2: Amplitude of the wakefields in the ranges $\xi_0 = 7 \pm \frac{\lambda_{pe}}{2}$ cm (a) and $\xi_0 = 3 \pm \frac{\lambda_{pe}}{2}$ cm (b), 3 cm behind each seed, normalized to the amplitude in the same ξ-region at $z = 50$ cm, which is the distance at which the seed electron bunches already left the simulation window. The curves start at $z = 50$ cm to compare the SM growth excluding the fields driven by the seed electron bunch.

d]). The larger growth leads to an earlier formation of microbunches behind the seed electron bunch whereas the bunch front is starting to modulate, but has not yet reached saturation, as observed in the growth after $z = 3$ m in Fig. 2 (b). This difference in modulation is seen in Fig. 3 (a-c). The transverse wakefields at this z-position are still shifted by $\approx \lambda_{pe}/2$ and are continuously growing from front to back, as the effect from the bunch front is still small. They have reached an amplitude of ≈ 200 MV/m, a consequence of SM.

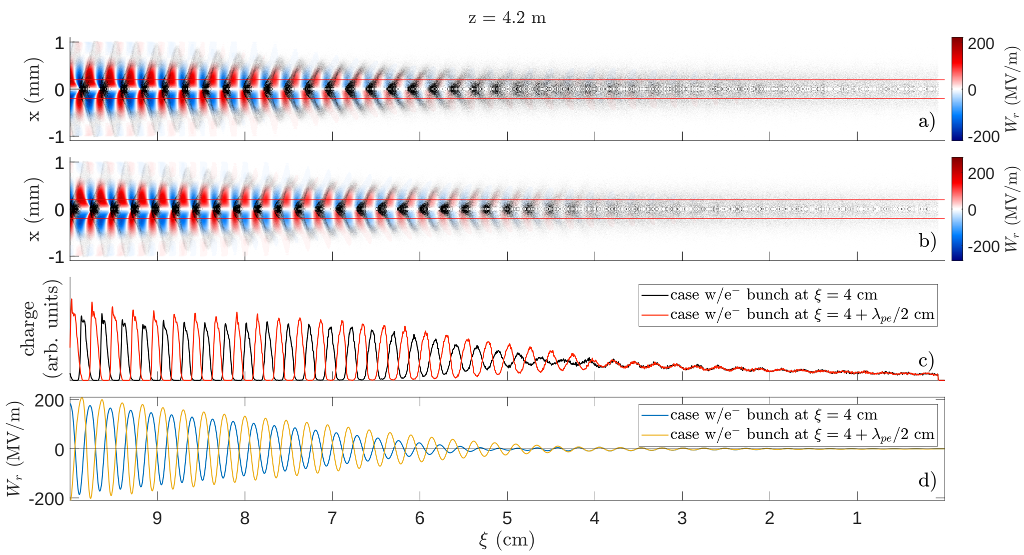

Figure 3: Same as Fig. 1, but $z = 4.2$ m.

In the first centimeters of propagation, the transverse fields driven by the unmodulated proton bunch are always defocusing for the electron bunch. As it propagates, it loses energy through its own wakefields, dephases with respect to the proton bunch, and is expelled out of the plasma at $z \approx 50$ cm. The amplitude of the wakefields driven by the modulating bunch front eventually becomes large enough to start affecting the microbunch at the initial electron bunch ξ-position, which in turn affects the one behind it, and eventually the entire microbunch train. The effect on the microbunch train depends on its relative phase with respect to the modulation and wakefields in the front. When the microbunches are in the focusing phase of the wakefields driven by the front, they continue propagating along the plasma. On the contrary, when they are in the defocusing phase, they are defocused and disappear. At $z = 10\,\mathrm{m}$ and after $\xi_0 \approx 6\,\mathrm{cm}$, the microbunches have been expelled in one case, shortening the bunch train, and remain in the other case (Figs. 4 [a-c]). This significant difference could be measured in the experiment. Figure 4 (a) shows the off-axis proton bunch density, that previously formed microbunches, in the defocusing wakefields. The lineout of the wakefields are in phase all along the bunch at $z = 10\,\mathrm{m}$ (Fig. 4 [d]), after the out-of-phase microbunches have been lost. The amplitude is also lower than in Fig. 3 because of the loss of charge of the microbunch train in both cases.

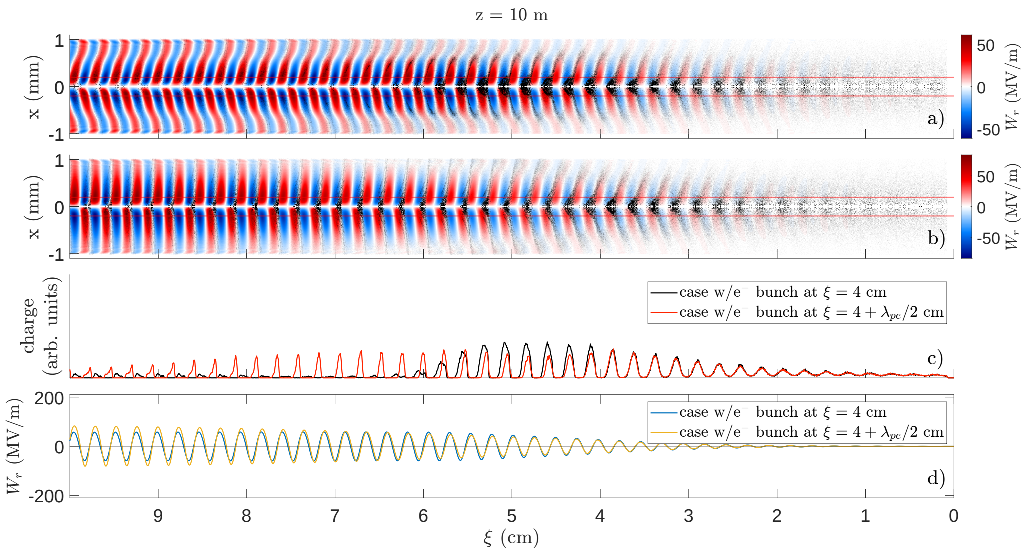

Figure 4: Same as Fig. 1, but $z = 10\,\mathrm{m}$.

Summary. We have shown that when using two seeds for the self-modulation, in the transition between seeding with the electron bunch or the density cut, a shift in position of the electron bunch can lead to the defocusing of microbunches from axis, shortening the microbunch train. This occurs because, behind the electron bunch, the modulation growth is larger and it saturates first, creating a microbunch train earlier in the propagation. Then the bunch front modulates and the wakefields it drives either expel microbunches from the axis or focus and maintain them until the plasma exit.

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