Fluid modelling of the weakly magnetized plasma column MISTRAL

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In a magnetized plasma column the presence of a radial electric field or a radial gradient of the plasma pressure leads to closed particle drifts in the azimuthal direction. This drift, when combined with plasma inhomogeneities, favors the emergence of instabilities that significantly boost transport across the magnetic field B (« anomalous transport »). Cross-field configurations are used in a variety of applications, including ions sources and Hall thrusters for satellites, magnetron discharges, Penning gauges and fusion plasmas. Understanding and ultimately controlling anomalous transport is therefore a crucial issue for these applications.

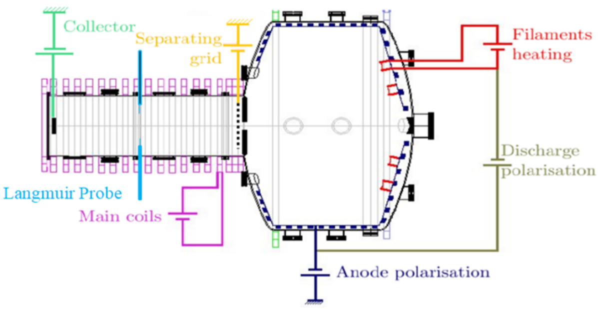

Figure 2: Schematic of MISTRAL

Variable	Notation	Definition	Values
Ion cyclotron frequency	ω_{ci}	eB/m$_i$	6.1 KHz
Ion thermal speed	v_{thi}	$\sqrt{K_B T_i/m_i}$	489 ms^{-1}
Ion larmor radius	ρ_i	m$_i v_{thi}/eB$	1.28 cm
Ion equilibrium angular frequency	ω_o	v_{i_o}/r	2.6 KHz
Density gradient scale length	$1/L_n$	$-n'_o/n_o$	0.69 m^{-1}
Equilibrium E×B angular frequency	ω_{Eo}	$(B\times\nabla\phi_o)/(rB^2)$	-11.13 KHz
Equilibrium ion diamagnetic angular frequency	ω_{*i}	$(B\times\nabla p_i)/(rn_o eB^2)$	-2.14 KHz

Table 1: Main variables with their definitions and values (at B = 16 mT and r = 2.5 cm)

MISTRAL is a linear magnetized plasma column based at the PIIM laboratory used to study ExB plasmas with magnetized electrons and weakly or not magnetized ions. MISTRAL is a versatile device in which the following typical plasma parameters can be achieved: plasma length L = 1m, plasma diameter = 8 cm, cylinderical vessel radius (a) = 10 cm, Te = 1-6 eV, ne = 10^{14}-10^{16} m^{-3}, B = 10-30 mT, P = 10^{-4}-10^{-2} Pa, Gas: H, He, Ar, Kr, Xe. The MISTRAL plasma has been characterized experimentally with several diagnostics (Langmuir probe, fast camera, emission spectroscopy). Coherent structures rotating in the azimuthal direction have been observed in MISTRAL rotating at a frequency comparable to the ExB rotation frequency with azimuthal wave number m = 1, 2 [1, 2]. A fluid model is in development to explain the coherent rotating structures observed in MISTRAL and MISTRAL like plasmas.

In MISTRAL, the observed rotating structures have a frequency comparable to the ion cyclotron frequency ω_{ci} and size comparable to the plasma size. The ion-neutral frequency ν_{in}, ExB frequency ω_E and diamagnetic frequency ω_* are all comensurable to ω_{ci}. The total plasma diameter is less than 10 larmor radii (ρ_i). So, an accurate treatment of MISTRAL plasma, therefore, requires to account for 1) ion polarisation effect ($\omega \approx \omega_{ci}$), 2) the radially global structure of instabilities, 3) the gyroviscous tensor (ion pressure gradient and finite Larmor radius effects), 4) ion-neutral collisions. No existing model so far meets these requirements. In [4], ion polarisation effects were accounted for, but the derivation was performed in the radially local limit and without considering the gyroviscous tensor. In [3], the gyroviscous tensor was included and the derivation is radially global but the ion polarisation is only partially accounted for (low frequency approximation $\omega << \omega_{ci}$). None of these works kept the ion-neutral friction. The goal of the present work is to progressively improve the model to be applicable to MISTRAL and MISTRAL like plasmas.

To start with, we consider a long cylindrical plasma bounded radially and immersed in the magnetic field which is constant along the radius and with axisymmetric geometry such that $\mathbf{B} = B\,\hat{e}_z$. The magnetic field is assumed to be constant along the radius. We assume no variations in the axial direction ($k_{||} = 0$). A dispersion relation is derived for a two species (electrons and singly charged ions) quasi-linear plasma in the electrostatic approximation for which the continuity equation for the species 's' is given by,

$$\frac{\partial n_s}{\partial t} + \nabla \cdot n_s \mathbf{v}_s = 0 \tag{1}$$

where s = i,e stands for ions and electrons respectively. For electron momentum equation, inertial effect, friction and pressure tensor are neglected,

$$0 = -\nabla\phi + \mathbf{v}_e \times \mathbf{B} - \frac{1}{en_e}\nabla p_e \tag{2}$$

For ions, the pressure tensor and friction is neglected,

$$\frac{\partial \mathbf{v}_i}{\partial t} + (\mathbf{v}_i \cdot \nabla)\mathbf{v}_i = \frac{e}{m_i}\left[-\nabla\phi + \mathbf{v}_i \times \mathbf{B}\right] - \frac{1}{m_i n_i}\nabla p_i \tag{3}$$

Using the perturbations of the form,

$$\begin{aligned} n(r) &= n_o(r) + n_1(r)\; exp[i(m\theta - \omega t)] \\ \mathbf{v}(r) &= \mathbf{v_o}(r) + \mathbf{v_1}(r)\; exp[i(m\theta - \omega t)] \\ \phi(r) &= \phi_o(r) + \phi_1(r)\; exp[i(m\theta - \omega t)] \end{aligned} \tag{4}$$

where subscript o indicates equilibrium quantities and linearising equations 1, 2, 3 using 4, the dispersion relation in the radially local limit ($\phi_1' = \phi'' = 0,\ \bar{n}_1' = \bar{n}_1'' = 0$), assuming quasi-neutrality is given by,

$$\bar{\omega}^3 - \left(\frac{m\bar{L}_n}{\bar{r}} + 3m\bar{\omega}_o\right)\bar{\omega}^2 + \left(3m^2\bar{\omega}_o^2 - 2C\bar{\omega}_o + \frac{m^2\bar{\omega}_o\bar{L}_n}{\bar{r}}(2+\bar{\omega}_o)\right)\bar{\omega} + \left(mC\bar{\omega}_o^2 - m^3\bar{\omega}_o^3\left(1+\frac{\bar{L}_n}{\bar{r}}\right) - \frac{m^3\bar{\omega}_o^2\bar{L}_n}{\bar{r}}\right) = 0 \tag{5}$$

where $C = 1 + 2\bar{\omega}_o$, $\frac{1}{\bar{L}_n} = -\frac{n_o'}{n_o}$ is the normalised density gradient and $\bar{\omega}_o$ is the equilibrium flow frequency which is given by, $\bar{\omega}_o^2 + \bar{\omega}_o - (\bar{\omega}_{*i} + \bar{\omega}_E) = 0$. Here, the frequencies and the lengths are normalised with the ion cyclotron frequency ω_c and ion larmor radius ρ_i respectively. This dispersion relation is exactly similar to the one given in [4] with only modification which is the pressure gradient term that enters the dispersion relation through the modification of $\bar{\omega}_o$ by $\bar{\omega}_{*i}$.

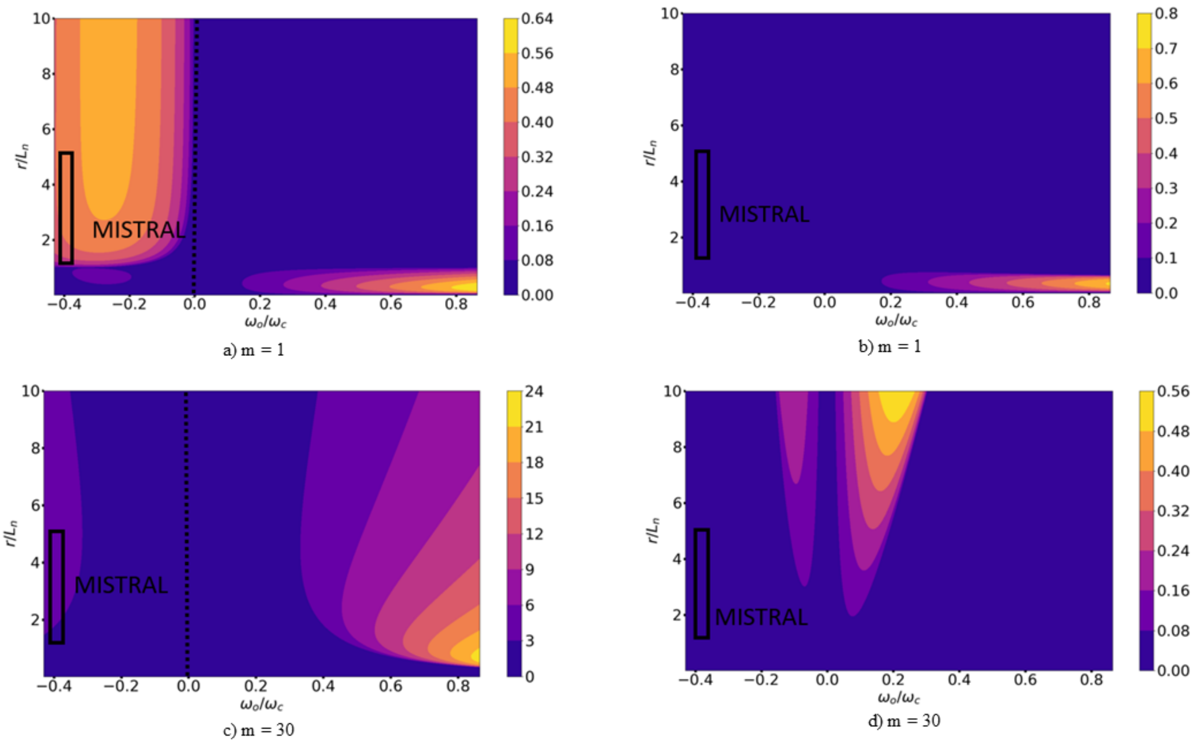

Figure 3: Stability diagram [4] as a function of normalised equilibrium flow frequencyω_o/ω_{ci} and dimensionless normalised density gradient r/L_n. Here the normalised growth rate γ/ω_{ci} has been plotted as a function of normalised equilibrium flow frequency ω_o/ω_{ci} and dimensionless normalised density gradient r/L_n for mode number m=1 and m=30. The black rectangular box indicates the range of MISTRAL parameters. Fig. 3(a,c) corresponds to the dispersion relation (5) and Fig. 3(b,d) corresponds to the dispersion relation (6)

In the low frequency (LF) assumption $\omega << \omega_{ci}$, the eq. (5) reduces to the quadratic dispersion relation given by,

$$\bar{\omega}^2 + \left(-2m\bar{\omega}_o + \frac{2rC\bar{\omega}_o}{m\bar{L}_n} - m\bar{\omega}_o^2\right)\bar{\omega} + \left(m^2\bar{\omega}_o^2 - \frac{\bar{r}C\bar{\omega}_o^2}{\bar{L}_n} + m^2\bar{\omega}_o^3\right) = 0 \qquad (6)$$

Stability diagrams : The stability picture obtained from eqs. 5 and 6 is shown in Fig. 3. At low azimuthal mode number m, the LF assumption tends to strongly decrease the mode growth rate in the region $\omega_o < 0$. At high m number, the unstable region for $\omega_o > 0$ in Fig.(3c), is stable in Fig.(3d) and also the growth rate is quite small which shows that at high m numbers, the LF assumption becomes quite important for high frequency values.

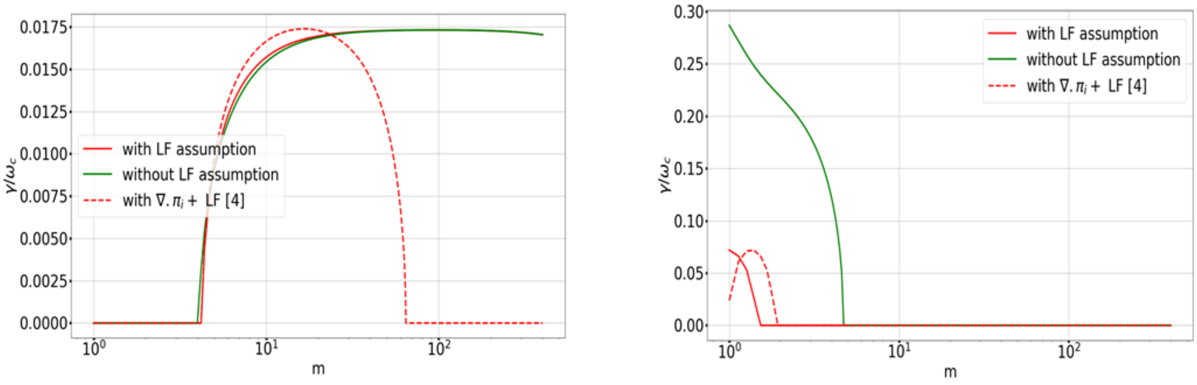

Figure 4: Growth rate versus m-number (a) $\omega_o/\omega_c = 0.0047$, $r/L_n = 18$ with $r = 3r_o$ (b) $\omega_o/\omega_c = -0.427$, $r/L_n = 1.34$ with $r = 2.5cm$

Difference between growth rates for different m numbers from the dispersion relation in the local limit : We studied the growth rates evaluated from Eqs. 5, 6 and using the dispersion relation (Eq. 25, 26) given in [3] but int the local limit (substituting radial derivatives equal to zero). Mainly, two cases were studied, 1) with the parameters used in [3] which are consistent with the LF assumption (Fig. 4(a)) and 2) MISTRAL parameters (Table I) which do not obey LF assumption (Fig. 4(b)). In Fig. 4(a), the growth rate evaluated by Eqs. 5, 6 (green and red curves respectively) is quite in agreement with the growth rate (dashed red curve) evaluated by the dispersion relation in [3]. But for high frequency regime (Fig. 4(b)), considerable difference can be seen between the growth rates computed from the dispersion relations with and without LF assumption and thus it points out to the need of models without LF assumption for MISTRAL like plasmas.

The inclusion of gyro-viscosity tensor ($\nabla.\pi_\mathbf{i}$) and ion-neutral collisions to have a complete radially global solution without LF assumption is currently in progress.

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