Models for modulated rotation of a Penning-trapped magnetized electron vortex in a variable-charge regime

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Nonneutral plasmas confined in Penning-Malmberg traps are used both as a very efficient means of studying collective-system dynamics - and especially fluid phenomena - and as a tool to further physics investigation, such as in the case of antimatter synthesis at very low energy for matter-antimatter symmetry measurements. In the context of basic collective phenomena, these plasmas are typically made of electrons generated by external sources and injected into the confinement region, which is defined by a series of cylindrical electrodes shaping an axial potential well within a uniform axial magnetic field. We have been using a quite unconventional production scheme, i.e. using a low-amplitude radio-frequency (RF) drive on one of the trap electrodes [1, 2]. When this RF drive is applied to one of the trap electrodes and its frequency is $\sim 1-20$ MHz (corresponding approximately to axial bounce frequencies of electrons for energies of few to some ten eV), it can effectively heat up the few free electrons in the residual gas and initiate a discharge and accumulation of an electron plasma. With respect to more conventional discharges, here RF power and residual gas pressure are much lower: RF amplitude is ≤ 10 V and pressure is in the high to ultra-high vacuum regime ($10^{-7}-10^{-9}$ mbar). Interesting stationary states are reached within some seconds where the electron plasma (contaminated by transient and partially-trapped ions) maintains total charge and spatial density distribution.

Among many configurations observed experimentally, which include diffuse as well as non-axisymmetric structures, a peculiar example is the creation of a single round column (vortex) of electrons displaying a radial displacement with respect to the longitudinal symmetry axis and thus rotating around it, with typical orbiting frequency $\omega_1/2\pi \sim 1-10$ kHz. Total charge ($\sim 0.1-1$ nC) and density profile (average density $5\cdot 10^6 - 5\cdot 10^7$ cm^{-3}) show very robust stability to perturbations and usual instabilities, e.g., ion-induced instability and the relevant loss of confinement, as long as the ionizing drive is maintained. Even more peculiar is the occurrence of stable low-frequency ($\sim 1-10$ Hz) oscillations of both charge and radial offset in these rotating vortices [3].

In this contribution, we discuss the interpretation of these oscillations. We consider only the dynamics of the electron vortex, and construct equations for the dynamics of offset, or orbit radius r, and line charge density $\lambda = Q_p/L_p$ (which is basically equivalent to the electron plasma

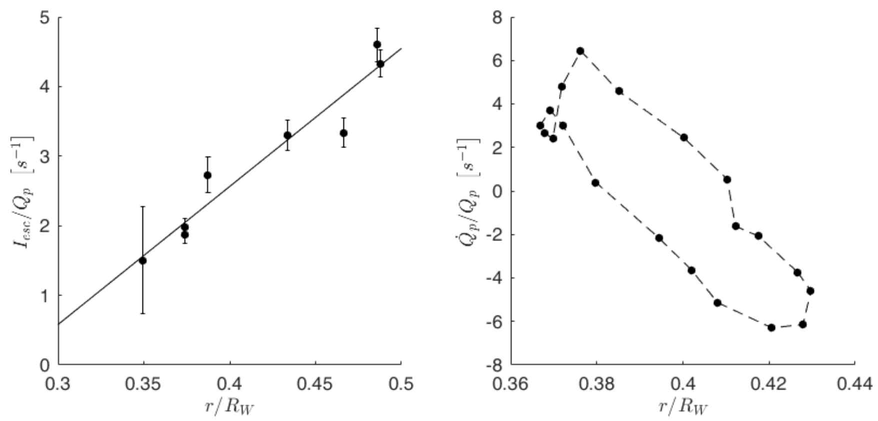

Figure 1: Left: Ratio of axial escape current to trapped electron column charge. Data are plotted against the respective orbit radius, normalized to the trap radius R_W. Different radii are obtained squeezing the orbit of the vortex by means of a static octupolar potential. Right: Normalized electron plasma charge variation versus the orbit radius along a charge-offset modulation period.

charge Q_p as the plasma length L_p is approximately constant; notice that the following treatment considers a generic positive λ). In the usual pressure conditions and at room temperature, number and density of neutrals are easily larger than those of electrons by two orders of magnitude, hence we neglect the dynamics of neutrals. We also ignore for the moment the variation of ion number and density, i.e. we approximate as constant the ionization rate and ion confinement efficiency – a cruder but initially acceptable assumption to simplify this preliminary treatment.

The line charge density equation proceeds from experimental observations. A RF-driven electron vortex rotating on a stable offset can be forced onto another orbit radius applying a static octupolar potential on a suitably split cylindrical electrode. We observe (Fig. 1, left) that the ratio between axial escape current (RF-heated electrons that overcome the trapping potential) and vortex charge increases with increasing offset, along a linear slope as long as the normalized displacement r/R_W, with R_W trap radius, is within $0.1-0.2$ (larger displacements do not yield reliable information, as significantly different equilibrium states are created). This suggests that the trapping efficiency of heated electrons decreases with increasing radius and thus the relative charge variation obeys, to a first-order approximation, a trend $\dot{\lambda}/\lambda \sim -r$. Although not a direct proof, this indication is confirmed *a posteriori* by measuring the relative time variation of trapped charge vs offset from plasma columns exhibiting the low-frequency modulation (Fig. 1, right). Since an equilibrium position r_0 must exist with charge balance and stable offset, we linearize around r_0 and finally write a line charge density dynamics equation in the form

$$\dot{\lambda} = \eta\,(r-r_0)\,\lambda, \quad \eta > 0 \tag{1}$$

As far as the dynamics of the electron column is concerned, we consider the two-dimensional motion only, in the cold-fluid $\mathbf{E}\times\mathbf{B}$ approximation, which holds since the transverse motion time scale is much larger than that of the axial bounce and the cyclotron motion is orders of magnitude faster and smaller in amplitude [4]. In this context, the motion of the column in the absence of perturbations such as additional external fields or ion contaminations would obey the equation $\mathbf{v}=\mathbf{E}\times\hat{e}_z/B$, where $\mathbf{v}$ is the transverse velocity of the vortex and $\hat{e}_z$ the unit vector in the longitudinal direction. Written in components, the equation reads $\dot{r}=0$, $r\dot{\theta}=-E_r(r)/B$, i.e. a constant-frequency rotation at stable offset. In our case the radial component is nevertheless affected by the presence of positive ions [5], who induce well-known instabilities leading to offset growth, i.e. transient-ion and trapped-ion instabilities, respectively accounting for ions lost after a single passage through the electron column [6] and ions bouncing multiple times across the confinement region due to the formation of a so-called nested potential well [7]. These two mechanisms yield algebraic and exponential offset growth, respectively, i.e. their combined effect can be written as $\dot{r}=\gamma r+D$, where $\gamma=\beta\omega_1=-\beta\dot{\theta}$ and β is proportional to the ion fraction. By substitution of the θ-component of the velocity equation into the radial one, we get $\dot{r}=\beta E_r(r)/B+D$. The electric field can be written as

$$E_r(r)\simeq\frac{\lambda r}{2\pi\varepsilon_0 BR_W^2}\left[1+\left(\frac{r}{R_W}\right)^2\right]-\tilde{T}r. \tag{2}$$

The first term represents the self-consistent electron vortex field written as the image-charge field, approximated for moderate offset and small plasma transverse size. The second term is the result of a ponderomotive (time-averaged) approximation of the fast-oscillation RF field, which contains both an axial component - not visible in the two-dimensional description, but implicitly considered as the source of ionization - and a radial component (we omit here the full calculation for the sake of brevity) [8]. Finally, redefining some constants we get the system of equations

$$\dot{\lambda}=\eta\left(r-r_0\right)\lambda \tag{3}$$

$$\dot{r}=k\lambda r\left[1+(r/R_W)^2\right]-Tr+D \tag{4}$$

which exhibits a Lotka-Volterra form. Two fixed points (λ_*, r_*) – where $\dot{\lambda}=0$, $\dot{r}=0$ – exist, i.e. $P_1=(0,D/T)$ (not physically interesting) and $P_2=\left((Tr_0-D)/\left[kr_0\left(1+(r/R_W)^2\right)\right],r_0\right)$. Calculating the Jacobian matrix in P_2 and its eigenvalues one can infer the stability properties of P_2. The eigenvalues read $\mu_{1,2}=1/2\left(\mathrm{TR}(J)\pm\sqrt{(\mathrm{TR}(J))^2-4\det(J)}\right)$, where $\mathrm{TR}(J)$ and $\det(J)$ are the trace and determinant of the Jacobian matrix, respectively. Oscillations occur when the argument of the square root $\Delta<0$, while damping requires $\mathrm{TR}(J)<0$. These two

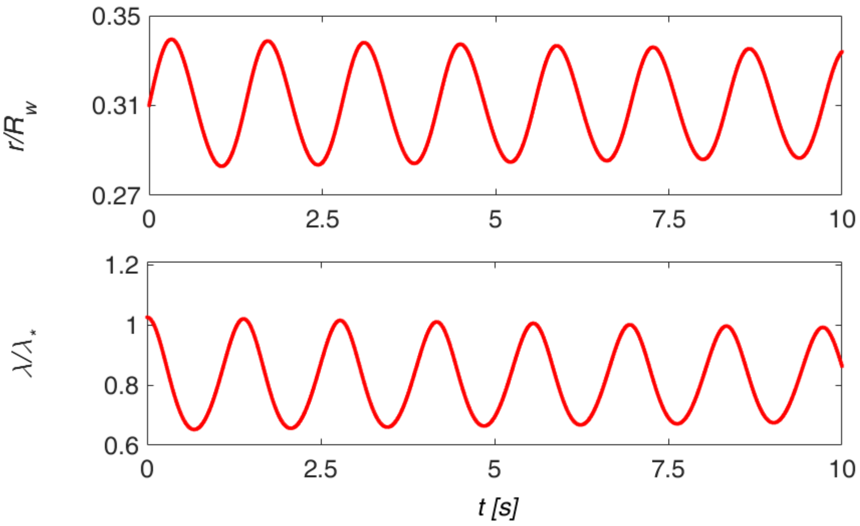

Figure 2: Offset- and charge-oscillating solution of eqs. 3 and 4 around the fixed point $(\lambda_*, r_*) = (0.92\text{ nC}, 0.31R_W)$. System parameters: $\eta = 1.2 \cdot 10^3\ \text{m}^{-1}{\cdot}\text{s}^{-1}$; $k = 2.2 \cdot 10^{10}\ \text{s}^{-1}$; $T = 2.2\ \text{s}^{-1}$; $D = 5 \cdot 10^{-3}\ \text{m}{\cdot}\text{s}^{-1}$.

conditions read, respectively

$$\left[\frac{2T\left(r_0/R_W\right)^2 - D\left[1+3\left(r_0/R_W\right)^2\right]/r_0}{1+\left(r_0/R_W\right)^2}\right]^2 < 4\eta\left(Tr_0 - D\right) \tag{5}$$

$$2T\left(r_0/R_W\right)^2 < \frac{D}{r_0}\left[1+3\left(r_0/R_W\right)^2\right] \tag{6}$$

The small-amplitude oscillation frequency is $\omega_{LF} = \sqrt{\eta\left(Tr_0 - D\right)}$. Realistic experimental parameters can satisfy these conditions. As an example, a numerical solution of the system of eqs. 3 and 4 is plotted in Fig. 2, showing a slowly damped oscillation of frequency around 1 Hz. We conclude that despite a complex dynamics spanning a wide range of time scales, this sketch can reproduce the main features observed in the experiments.

References

[1] B. Paroli, F. De Luca, G. Maero, R. Pozzoli and M. Romé, Plasma Sources Sci. Tecnol. **19**, 045013 (2010)

[2] G. Maero, S. Chen, R. Pozzoli and M. Romé, J. Plasma Phys. **81**, 495810503 (2015)

[3] B. Paroli, G. Maero, R. Pozzoli and M. Romé, Phys. Plasmas **21**, 122102 (2014)

[4] C. F. Driscoll and K. S. Fine, Phys. Fluids B **2**, 1359 (1990)

[5] G. Maero, Il Nuovo Cimento C **40**, 90 (2017)

[6] J. Fajans, Phys. Fluids B **5**, 3127 (1993)

[7] A. A. Kabantsev and C. F. Driscoll, Fus. Sci. Technol. **51**, 96 (2007)

[8] G. Maero, B. Paroli, R. Pozzoli and M. Romé, Phys. Plasmas **18**, 032101 (2011)