Effect of density ramp on electron acceleration driven by tightly focused ultrashort laser pulses

Zs. Lécz[1], A. Andreev[1], C. Kamperidis[1], N. Hafiz[1]

[1] *ELI-ALPS, Szeged, Hungary*

The envelope equation describing the evolution of the Gaussian laser spot size in the paraxial approximation is written in the following form [1]:

$$\frac{d^2w}{dx^2} = -2\frac{K(w,x)}{w} + \frac{4}{k_0^2 w^3}, \tag{1}$$

where $k_0 = 2\pi/\lambda_0$ is the laser wave number, $K(w,x)$ is a radially averaged quantity and it is proportional to $rd[n_e/(\gamma n_c)]/dr$, where n_e is the electron density in the plasma, $n_c = \omega_0^2 m_e \varepsilon_0/e^2$ is the critical density, defined by the laser wavelength $\omega_0 = 2\pi c/\lambda_L$, and $\gamma = \sqrt{1+a_0^2/2}$ is the averaged relativisitc Lorentz factor of electrons inside the laser field. The right hand side of Eq. (1) decides the evolution of the laser spot radius (w): if the first term (plasma term) is stronger then relativistic self focusing takes place and the spot size decreases, while in the case of diffraction the second term is dominant and the spot size increases. Since the laser field amplitude depends on w ($a_0 \propto 1/w$), the relativistic focusing term will be proportional to $K \propto w^{-2}$, thus it gets weaker as the pulse is diffracted. In order to compensate this an increasing density profile is needed.

One can understand the pulse evolution in a plasma by analyzing the radial dependence of the phase velocity of the laser field. The effective phase velocity of a laser pulse inside the plasma is given by [2]:

$$\frac{v_f}{c} = \left[1 - \frac{k_p^2}{2k_0^2} - \frac{2}{k_0^2 w^2}\left(1 - \frac{r^2}{w^2}(1-\alpha^2)\right)\right]^{-1}, \tag{2}$$

where $\alpha = x/L_R$, $L_R = \pi w_0^2/\lambda_0$ is the Rayleigh length, $k_p = \omega_p/c$ is the plasma wave number with $\omega_p = \sqrt{e^2 n_e/(m_e \varepsilon_0)}$. In the limit of pure vacuum ($k_p = 0$) this expression takes the form presented in Ref. [3], with $w_{vacuum} = w_0\sqrt{1+\alpha^2}$. The evolution of the laser spot size, and intensity, is governed by the wavefront curvature, which is just the integral of the phase velocity along the longitudinal coordinate at each radial point. By using the replacement $k_p^2/(2k_0^2) = n_e/(2n_c\gamma)$ in Eq. (2) one can integrate the phase velocity with arbitrary longitudinal density profiles. For simplicity we assume only weak relativistic effects and the laser spot radius is approximated with the usual Gaussian expression: $w \approx w_0\sqrt{1+\alpha^2}$ and the γ factor depends on x via $\gamma = \sqrt{1+a^2/2}$, where $a = w_0 a_0/w$. In Fig. 1 the wavefront shapes are shown of a laser

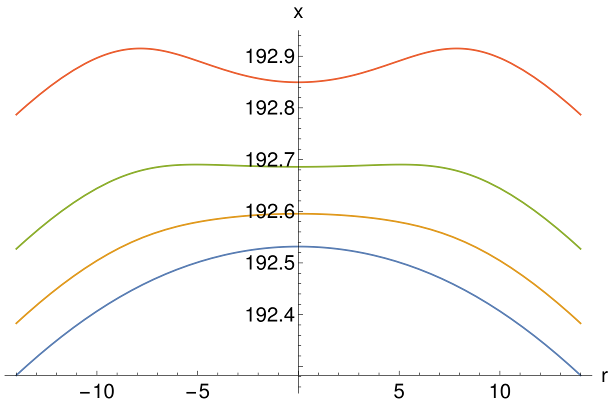

Figure 1: *Shape of the wavefront after one Rayleigh length propagation of the pulse with* $a_0 = 2$ *and* $w_0 = 7\,\mu m$ *in plasma with density:* $n_e/n_c = 1.5 \cdot 10^{-4}$ *(blue),* $n_e/n_c = 0.0012$ *(orange),* $n_e/n_c = 0.0027$ *(green) and* $n_e/n_c = 0.0054$ *(red).*

pulse focused to 7 μm waist radius, which propagates L_R distance in a uniform plasma with different densities. Initially the wavefront is constant in r and it is equal to zero. One can see that there is a plasma density value at which the wavefront curvature is negligible, at least in the interval $-w_0 < r < w_0$, which means that the central part of the laser pulse does not diffract. At lower density the pulse diffracts almost as in vacuum ($dw/dr < 0$), while if the density is high enough self focusing is the dominant ($dw/dr > 0$). Similar result can be obtained by choosing a spatial dependence of $n_e(x)$, that can resemble a Gaussian gas jet, for instance.

For a more precise modeling of the pulse evolution we need to solve numerically Eq. (1), where the initial spot radius and its derivative (dw/dr) can be also defined in order to use the focus position (x_f) as an input parameter. It is important to note that in the case of few-cycle pulses the radial density profile witnessed by the laser field is proportional to the laser envelope, i.e. it has a Gaussian distribution due to the longitudinal ponderomotive force. This density modulation we define as $\delta n = n_0[1 + f exp(-r^2/w^2)]$, where $f < 1$ is a small number expressing the amplitude of the density modulation and, according to our simulations, its value is between 0.3-0.6 for a_0 between 2 and 3. In the case of a gas jet the plasma density profile is given by the following function $n_e(x,r) = n_0 \exp(-(x-x_0)^2/L_R^2)[1 + f exp(-r^2/w^2)]$, where x_0 is the center of the gas jet. This density function is used in Eq. (1) in the expression of K [1]:

$$K(w,x) = -\left[\frac{\omega_p(x)}{ck_0}\right]^2 \int_0^{R_c} \frac{4r}{w^2}(1-s)e^{-s}\frac{n_e(r)}{n_0}\left(1+\frac{e^{-s}}{2W^2}\right)^{-1/2} dr, \qquad (3)$$

where $s = 2r^2/w^2$, $W = w/(w_0 a_0)$ and the $x-$dependence of the density is included in the plasma frequency.

The solutions of Eqs. (1, 3) are shown in Fig. 2, where $E_L = E_{L0} w_0/w$ is plotted versus

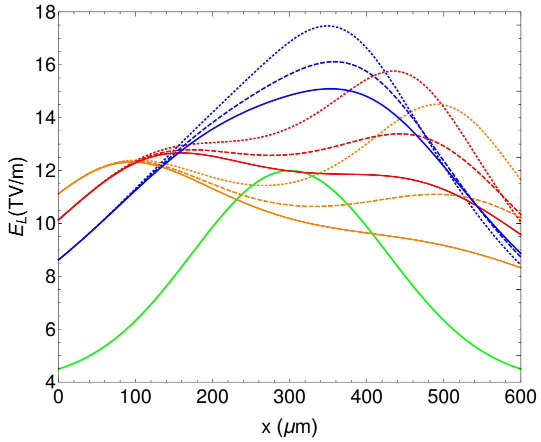

Figure 2: *Results of modeling by solving numerically Eq (1). Here* $w_0 = 7\,\mu m$ *and* $a_0 = 3$. *The green curve is the density profile of the gas jet. The focusing position is varied:* $x_f = 80\,\mu m$ *(orange),* $x_f = 120\,\mu m$ *(red),* $x_f = 180\,\mu m$ *(blue). The dotted lines are obtained for* $f = 0.0$, *dashed lines are for* $f = 0.3$ *and the full lines for* $f = 0.5$.

distance. Here the peak density is $n_0 = 1.2 \times 10^{19}$ cm^{-3}. Self-focusing is the strongest when the pulse is focused deeper inside the gas jet, but the spot size remains almost unchanged within the gas jet if the focus position is farther behind the center of the jet. The effect of the density modulation is also clearly visible: if f is larger (stronger ponderomotive repulsion), then the self focusing is weaker, but the spot size still changes slowly as the pulse passes through the plasma (see full lines).

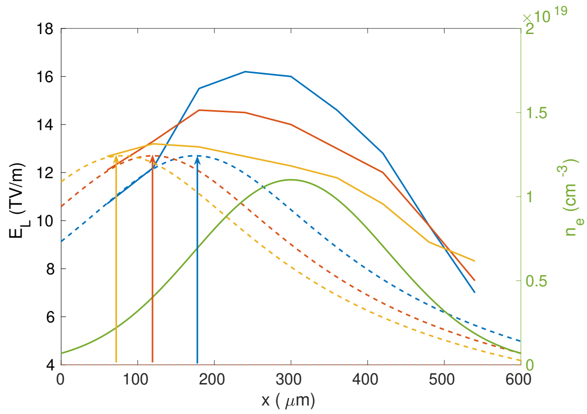

Figure 3: *Results of PIC simulations with parameters detailed in the text (full lines). The dashed lines show the laser field amplitude in vacuum propagation. The focus positions and other physical parameters are the same as in Fig. 2.*

We have tested the predictions of the numerical modeling by performing particle-in-cell simulations [4], where all nonlinear effects are included. The density profile in the simulation is the same as in Fig. 2, the laser intensity is $I_L = 2 \times 10^{19}$ W/cm^2 and its duration is $t_L = 8$

fs (FWHM). The size of the simulation box (moving window) is $24 \times 30 \times 30\,\mu\mathrm{m}^3$, which is resolved by $600 \times 300 \times 300$ grid points, where the largest resolution is applied along the propagation ($x-$)direction. The laser field amplitude (shown in Fig. 3) evolves very similarly to the model result (Fig. 2). We find better agreement with the model if $f \approx 0.5$, which is close to the value measured in the simulation $f = 0.42$. In the simulations the pulse starts to diffract faster than in the modeling, which can be attributed to the energy depletion. This effect is not included in Eqs. (1, 3) and the rate of energy loss is not trivial in the case of few-cycle pulses, it is lower compared to the widely used scaling formula [5], which gives a depletion length $L_d \approx ct_L n_c/n_e$. If we consider 2 times longer depletion length for few-cycle pulses the diffraction compensation can be effective only if $L_d > L_R$, which implies: $n_e/n_c < 2N_L\lambda_0^2/(\pi w_0^2)$, where N_L is the number of laser cycles within the pulse. For efficient wakefield generation one can set the conditions $w_0 \approx \lambda_p\sqrt{\gamma}/2$, which gives a lower limit for the pulse duration: $N_L > \pi\gamma/4$.

It has been shown that the vacuum diffraction (or Rayleigh diffraction) of a tightly focused ($w_0 < 10\,\mu$m) laser pulse can be compensated by choosing a plasma refractive index which changes the convex shape of phase velocity and results in a flattop wavefront even after significant propagation inside the plasma. It is found that the laser pulse has to be focused in front of a gas jet in order to reduce the strong variation in laser spot radius, which results in a more stable wakefield structure and a low divergence laser pulse at the rear side of the gas jet. This allows us the further use of the pulse in a second acceleration stage, which can be a second gas jet or a discharge capillary waveguide. The scheme works even for few cycle pulses, but the required pulse duration is proportional to the laser field amplitude, thus at higher intensity longer pulses are needed.

References

[1] P. Sprangle et al.,IEEE Transactions on Plasma Science **15**, 145 (1987)

[2] E. Esarey et al., JOSA B **12**, 1695 (1995)

[3] B S Davis and L Kaplan, Journal of Optics **15**, 075706 (2013)

[4] T. D. Arber et al., Plasma Phys. Control. Fusion **57** 113001 (2015)

[5] W. Lu et al., PRST AB **10**, 061301 (2007)