Quasi-static modeling of two-dimensional adiabatic compression of FRC plasmas

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Introduction

Adiabatic compression of field-reversed configuration (FRC) is among the promising paths to the compact, economical neutron sources and potential fusion reactors. The FRC equilibrium is essentially two-dimensional, however, previous theory models for FRC adiabatic compression often fail to take into account of the two-dimensional (2D) spatial and geometric features of FRC equilibrium. In this work, key scaling laws of the 1D quasi-static adiabatic compression of FRC plasma [1] are amended from 2D FRC MHD equilibriums numerically obtained using the Grad-Shafranov equation solver NIMEQ [2-3]. Based on the conservations of magnetic flux and entropy density, the quasi-static variation of the plasma pressure profile is obtained in the highly elongated FRC regime in a 1D approximation, which is then further used to construct the corresponding 2D FRC equilibriums and scaling laws for the quasi-static adiabatic compression of FRC. The amended 2D scaling laws are applied to the estimate of the upper limits of stable adiabatic compression ratio along with the empirical stability criterion for FRC.

FRC equilibrium during quasi-static adiabatic compression

The pressure profile inside the separatrix of an FRC plasma during compression can be modelled as $P(\psi) = P_m\beta(\phi)$, where $\beta(\phi) = \frac{\phi+\sigma\phi^2}{1+\sigma}$ to classify a long equilibria, $\phi = \psi/\psi_t$, P_m is the maximum value of pressure profile, σ is the compression ratio, ψ is the poloidal flux, and ψ_t is the magnetic flux inside the separatrix with $\psi_t = \psi_{sep} - \psi_{axis}$ [1]. The compression ratio is defined as $\sigma = R_s/R_W$, for which R_s is the radius of the separatrix and R_W is the wall major radius.

Let $\Psi = (\psi/\psi_t) + (1/2\sigma)$, thus the G-S equation for the FRC plasma inside the separatrix becomes $\Delta^*\Psi = -\mu_0 R^2 \frac{2P_m\sigma}{(1+\sigma)\psi_t^2}\Psi = -dR^2\Psi$, where $d = \frac{2\mu_0 P_m \sigma}{(1+\sigma)\psi_t^2}$. For any specific radial compression ratio and P_m scaling, the 2D FRC equilibrium is numerically solved using NIMEQ for the set of the external coils chosen to ensure that the σ at $Z = 0$ from the NIMEQ solution matches the σ in the pressure profile.

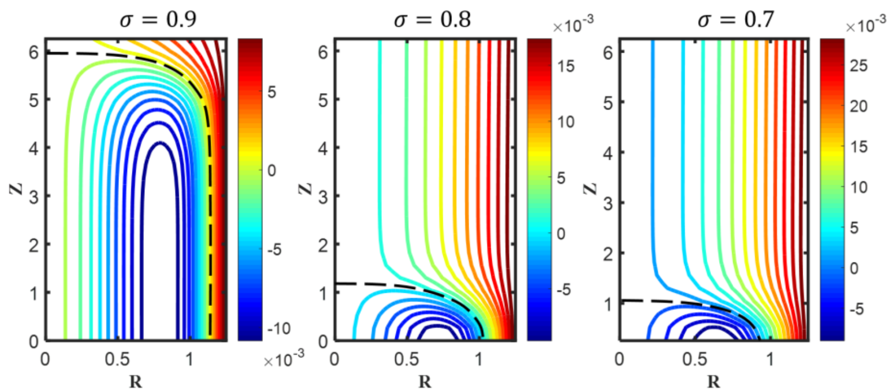

Figure 1. The contour of magnetic fluxes of 2D MHD equilibrium compression for different radial compression ratio, with initial FRC length $l_i = 13m$, $n_{i0} = 3 \times 10^{19}\ m^{-3}$, $\mu_0 P_{mi} = 0.0018\ Pa$, and the initial magnetic field at the wall $B_0 = 1.0T$.

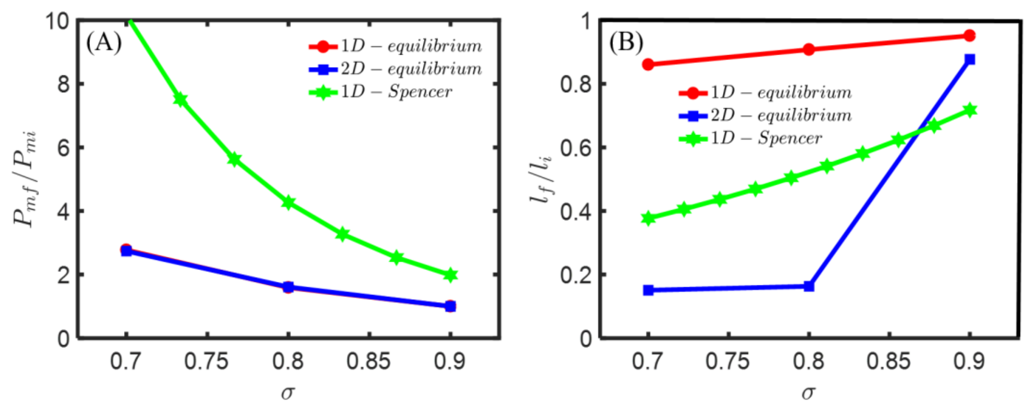

Figure 2. (A) The maximum final pressure and, (B) separatrix length as a function of radial compression ratio measured from the 1D equilibriums, 2D equilibriums and 1D Spencer scaling law.

Applying the analytical adiabatic compression scaling law based on 1D approximation [1], we have $P_m = P_{mi}\sigma^{-2(3-\epsilon)}$, where P_{mi} is the initial maximum pressure value before compression, and $\epsilon = -0.25$ in our case. From the corresponding 2D FRC equilibrium, we find that, as the initial elongation increases, the shrinking of FRC along the axial direction during compression becomes faster than the prediction from the 1D Spencer scaling law [4]. This is likely because the entropy density is not well conserved among those sequence of 2D FRC equilibriums with different compression ratio σs.

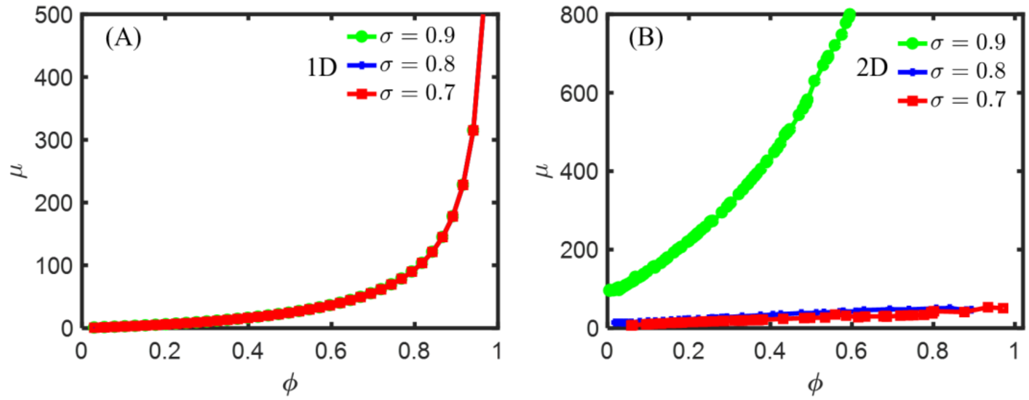

Figure 3. The $\mu(\phi)$ profiles calculated from (A) 1D approximation and (B) 2D NIMEQ solution for different radial compression ratio.

One-dimension equilibrium calculation are made to obtain pressure profile that will produce 2D equilibria. Therefore, for better entropy conservation, the 1D FRC equilibrium solver is applied to the iteration to obtain the 1D equilibria with fixed entropy profiles for the highly elongated FRC. The numerical $P(\psi)$ profile and l_s thus derived is further applied to the numerical solution of the 2D FRC equilibrium using NIMEQ with matching compression ratio σ. The equilibrium poloidal flux contours from this method for the adiabatic compression process are given in Fig. 1. The corresponding scaling laws for the final pressure and the separatrix length show that during the compression process, the FRC shape from 2D equilibrium calculation shrinks faster axially than the 1D Spencer scaling law prediction (Fig 2). For the entropy per unit flux $\mu(\psi) = P(\psi)\left(2\pi \oint \frac{dl}{B}\right)^{\gamma}$, its 1D approximation for the elongated FRC is based on $\oint \frac{dl}{B} \approx \frac{2l_s}{B} = \frac{2l_s}{\{2[P_m - P(\psi)]\}^{1/2}}$, where $\gamma = 5/3$ and l_s is the length of FRC separatrix. We compare the $\mu(\psi)$ profiles calculated from the 1D and the 2D equilibrium calculations. The local entropy density conservation among various compression ratio from the 1D iterative calculation is indeed much improved in comparison to the results based on the analytical 1D scalings [4], however, such an entropy density conservation is not maintained in the 2D equilibrium calculations (Fig. 3).

FRC stability criterion

Based on experimental data, an empirical stability criterion of FRC can be written as $S/_{\kappa} < 3.5$, where $\kappa = Z_s/R_s$ is the elongation of the separatrix, $S = R_s/\delta_i$ is the ratio of the radius of the separatrix to the ion skin depth, $\delta_i = c/\omega_{pi}$, and $\omega_{pi} = \sqrt{n_i e^2/\epsilon_0 m_i}$ [6-7]. From these we have $S/\kappa = \frac{R_s/\delta_i}{Z_s/R_s} = \frac{R_s^2}{Z_s c}\sqrt{\frac{n_i e^2}{\epsilon_0 m_i}}$, which is evaluated directly using both the 1D and 2D MHD equilibrium solution for each given σ and the magnetic field strength at the wall B_w, where the FRC length is measured from the separatrix and the n_i is calculated from $n_i = \frac{N_i}{V} = \frac{N_i}{\pi R_s^2 l_s} = \frac{N_i}{\pi\sigma^2 R_w^2 l_s} = \frac{n_{i0} l_i}{\sigma^2 l_s}$ (Fig. 4). The black dashed line signifies the boundary where S/κ equals to 3.5. Below this line is the stable regime for FRC adiabatic compression. The measured estimate for S/κ from the 1D and 2D MHD equilibrium solutions amends the prediction from the 1D analytical scaling law about the stable FRC compression limit.

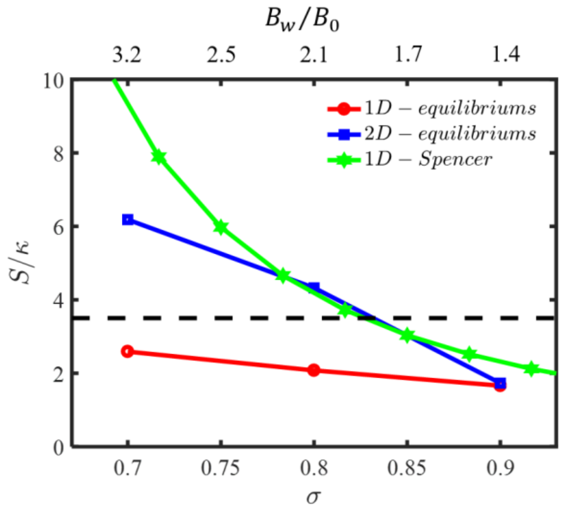

Figure 4 The S/κ stability criterion calculated from 1D equilibriums, 2D MHD equilibriums, and 1D Spencer scaling law for different radial compression ratio σ and magnetic field strength at the wall B_w.

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