Concepts for studying strong-field QED using laser-electron colliders

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Introduction

High-intensity laser facilities in conjunction with electron accelerators can probe extreme domains of radiation reaction (RR) and other effects of strong-field quantum electrodynamics (SFQED) [1, 2, 3]. Effects can be quantified by the dimensionless acceleration of electrons in their rest frame caused by laser radiation

$$\chi = \gamma \frac{\sqrt{\left(\vec{E} + (\vec{v}/c) \times \vec{B}\right)^2 - \left(\vec{E} \cdot \vec{v}/c\right)^2}}{E_{crit}}, \tag{1}$$

where γ is the Lorentz factor, $\mathbf{v}$ the velocity and $\mathbf{E}, \mathbf{B}$ are the electromagnetic fields at the location of the electron. Additionally, c is the speed of light and E_{crit} is the Schwinger field given as $E_{crit} = m^2c^3/(e\hbar)$ with m, e being the electron mass and charge respectively and $\hbar$ is the reduced Planck constant. Moderate values ($\chi \gtrsim 1$) delimit the quantum regime of RR whereas extreme values ($\chi \gtrsim 1600$) demarcate the conjectured breakdown of perturbative SFQED [4, 5, 6]. Thus, probing these effects require large values of χ in experiments using extensive electron beam energy $\varepsilon = mc^2\gamma$ and laser input power P. Ample ways to accelerate electrons exists but the use of advanced focusing of the laser pulse other than tight-focusing is not widely discussed in literature (exceptions are [7, 8]). One example of this is the Dipole wave [9, 10] which maximize either fields $\vec{E}, \vec{B}$ in the focal region, directly related to the increase of χ. However, we will show that this is not the same as maximizing χ itself.

Maximization Problem

In 1986 Basset proved using multipole expansion that the dipole component provides the largest electromagnetic energy density for a given power P and that the powers of the components are additive [9]. In other words, the total power of the sum of components is the sum of powers for each component. As previously stated, the Dipole wave only maximize either the electric (electric Dipole wave) or the magnetic field (magnetic Dipole wave) at focus. To maximize χ, an analogous procedure to that of Basset can be adopted.

We assume that the value of χ reaches its maximum in the center of a spherical coordinate system $(x, y, z) \rightarrow (r, \theta, \phi)$ and that the electric field is pointing toward $\theta = 0$ (along z). Fur-

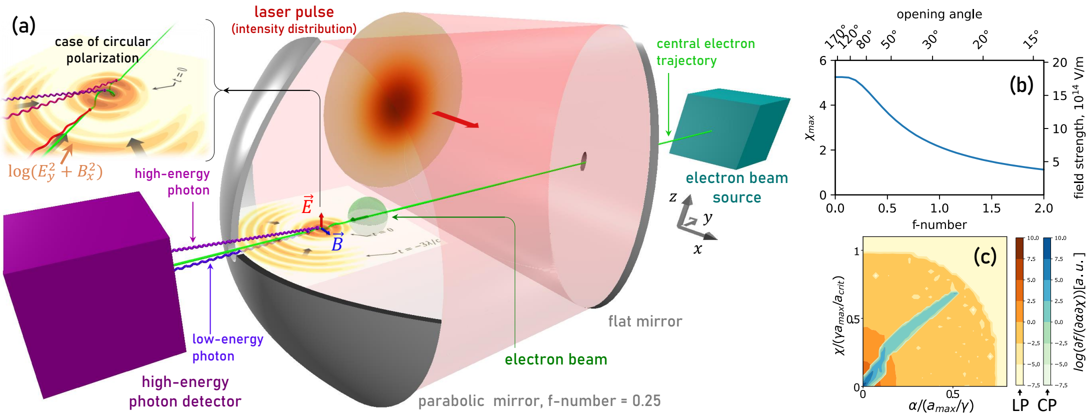

Figure 1: (a) schematic representation of a possible experimental setup with an insert showing the case of circular polarization (deviation angles are exaggerated); (b) $\chi_{\max}$ as a function of f-number; (c) correlation between deviation angle and χ.

thermore, the field can be expressed as the real part of complex fields $\vec{E}^\chi, \vec{B}^\chi$ multiplied with a factor $e^{-i\omega_0 t}$ where ω_0 is the frequency of the laser. If the complex fields are expanded in multipoles, the Maxwell equations take the form [11]

$$\begin{aligned}
E_r^E &= l(l+1)r^{-1} j_l(kr) Y_l^m(\theta,\phi), \\
E_\theta^E &= r^{-1}\partial_r\left(r j_l(kr)\right)\partial_\theta Y_l^m(\theta,\phi), \\
E_\phi^E &= im\sin^{-1}\theta r^{-1}\partial_r\left(r j_l(kr)\right) Y_l^m(\theta,\phi), \\
H_r^E &= 0,\ H_\theta^E = km\sin^{-1}\theta\, j_l(kr) Y_l^m(\theta,\phi), \\
H_\phi^E &= ikj_l(kr)\partial_\theta\left(Y_l^m(\theta,\phi)\right), \\
\vec{E}^B &= -\vec{B}^E,\ \vec{B}^B = \vec{E}^E.
\end{aligned} \tag{2}$$

Here, $\ell = 1,2,3$, $m = -\ell, -\ell+1, ..\ell$, $k = \omega_0/c$, $j_l(kr)$ and $Y_l^m(\theta,\phi)$ are spherical Bessel functions and spherical harmonics respectively. Subscripts indicate vector components and superscripts E, B denote electric and magnetic multipole components. In the limit $r \to 0$, only six components contribute to the field

$$\vec{E}^E_{1,-1} = \vec{B}^B_{1,-1} = k\,(6\pi)^{-1/2}\left(\hat{\vec{x}} - i\hat{\vec{y}}\right), \tag{3}$$

$$\vec{E}^E_{1,0} = \vec{B}^B_{1,0} = k\,(3\pi)^{-1/2}\hat{\vec{z}}, \tag{4}$$

$$\vec{E}^E_{1,1} = \vec{B}^B_{1,1} = -k\,(6\pi)^{-1/2}\left(\hat{\vec{x}} + i\hat{\vec{y}}\right), \tag{5}$$

where $\hat{\vec{x}}, \hat{\vec{y}}$ are the unit vectors along the x- and y-direction respectively. By the assumption that $\vec{E}$ is oriented along z, this has to be composed solely of eq. (4). Moreover, the magnetic field can be chosen to lie in the $xz-$plane so that it is formed by (4) and a combination of (3) and (5). By the additive power property of multipole components, the resulting field can be split into three parts. Given a input power P, a fraction aP will be delivered by $\vec{E}^{E}_{1,0}$, bP by $\vec{B}^{B}_{1,0}$ and the remaining part $(1-a-b)P$ by the combination $\sqrt{2}\left(\vec{B}^{B}_{1,-1}-\vec{B}^{B}_{1,1}\right)$ (factor of $\sqrt{2}$ ensures simultaneous peaking). It should be noted that the Lorentz force is maximized if the electron propagates in the $y-$direction. In this case, the absolute value of the force and the χ value becomes

$$\chi \propto \left(\left(a^{1/2}+(1-a-b)^{1/2}\right)^2+b\right)^{1/2}. \tag{6}$$

The choice $a=b=1/2$ will yield the maximum of eq. (6), denoted $\chi_{\max}$, meaning that the optimal geometry is formed by the sum of an electric and a magnetic dipole wave which we refer to as a "Bi-Dipole wave".

The formation of a Bi-Dipole wave is done by reflecting a seed pulse on a parabolic mirror illustrated in fig. 1. The seed should have linear polarization everywhere and the following intensity distribution [12]

$$I^p(R) \sim \left((R/(2L))^2+1\right)^{-4}. \tag{7}$$

Here, L is the distance to the mirror and R is the distance to the z axis in transverse plane. Also shown in fig. 1 is the dependence of $\chi_{\max}$ as a function of f-number from a finite parabolic mirror.

Experimental Strategies

To quantify emissions at high values of χ it is important to outline which particle or metric to employ. Tracking electrons by their energy spectrum and deflection angles will not be informative since single high χ emissions are indistinguishable from many low χ emissions. Secondly, due to the oscillatory structure of the laser field, the net deflection angle of an electron is cancelled. On the other hand, the instantaneous deflection angle in the strong-field region is large and any emitted photon will retain this trajectory. Moreover, photons have a longer mean free path in terms of the propagation length before pair-production as compared to that of electron emission of hard photons. Thus, tracking *primary photons* is a viable strategy. Primary photons refer to any photon being emitted by an electron at a large value of χ which in turn has not lost more than 1% of its initial energy prior to the emission. Using this definition we introduce the *signal-to-noise* ratio

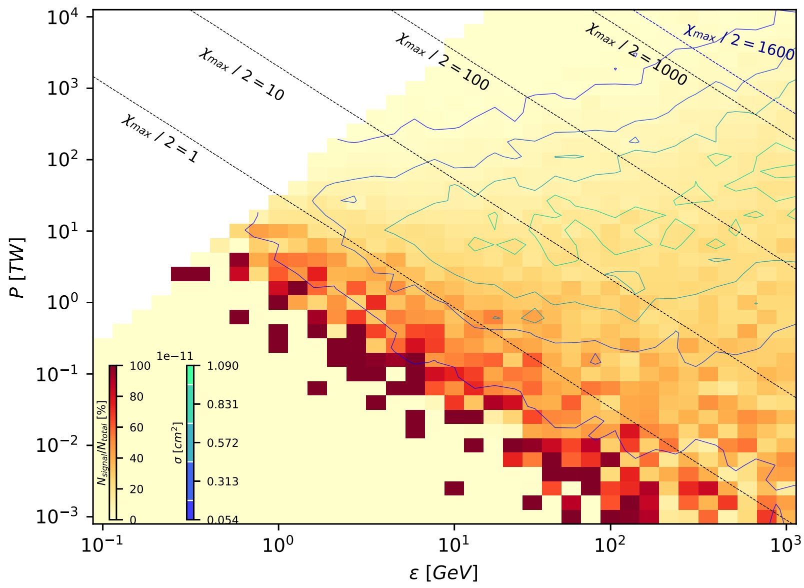

Figure 2: Signal-to-noise ratio and cross-section for a linearly polarized Bi-Dipole wave with $\lambda = 0.8\mu$m, $\tau_{\text{laser}} = 5$fs and $N_e = 10^5$ sampled at 36×36 points in the space of laser powers and electron energies. White regions demarcate non-sampled regions where QED cascade is suppressing the signal and dashed lines indicate contours of $\chi_{max}/2$.

$$\frac{N_{\text{signal}}}{N_{\text{total}}} := \frac{\text{\#Of primary photons}}{\text{\#Of primary and secondary/background photons}} \tag{8}$$

and the cross-section

$$\sigma := \frac{N_{\text{signal}}}{c\tau_{\text{laser}} n_e} \tag{9}$$

where τ_{laser} is the pulse duration and n_e is the number density of electrons.

Results of Simulations and Conclusions

Simulations shown in fig. 2 indicate that the Bi-Dipole wave can generate moderate signal-to-noise ratios for laser-beam parameters producing $\chi_{\text{max}}/2 \sim 10$. Note that we need not the ratio to reach unity since a Bayesian analysis comparing many experimental data with that of theoretical predictions can combat this. In conclusion, we found here the optimal focusing geometry for laser radiation to produce the highest possible values of χ. Moreover, we discussed how a clean signal of high χ emissions can be extracted and provided a parameter scan for laser-matter collider capabilities.

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