Inference of physics state variables from simulations − A new paradigm?

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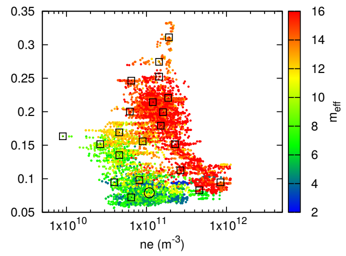
Introduction

For nearly a century, Langmuir probes have been the instrument of choice for measuring basic state parameters of plasma, such as the density, temperature and plasma potential. In that period numerous experiments and theories have been reported, with the objective of constructing better inference techniques for this relatively simple instrument. The number of studies made speaks to the importance of this diagnostic tool in lab, and more recently, in space plasma. The continuing work on this topic also speaks to the difficulty of accurately interpreting measurements made with this instrument in terms of physical parameters. The challenge stems from the fact that Langmuir probes don't directly measure the physical parameters of interest, but rather currents as a function of bias voltages. The determination of physical plasma parameters of interest, such as density and temperature, therefore requires models which relate probe current-voltage characteristics to plasma state variables, in order to solve this inverse problem. Until now, the only models used in practice, were based on analytic expressions obtained from theory, in which simplifying assumptions were made, in order to be analytically tractable. In recent years, however, sophisticated computer models have been developed to calculate probe characteristics with more physical processes, and more realistic geometries than what can be accounted for analytically. With such models, the direct calculation of probe characteristics, given plasma and satellite state variables, is relatively straightforward. In principle, the inverse problem consisting of inferring plasma state variables from characteristics, could be solved iteratively by carrying out simulations while fitting assumed plasma and satellite conditions, to measured characteristics. This approach is however not practical in actual data analysis, due to the considerable computing resources required in simulations. The solution explored here consists of using computer simulations to construct "solution libraries" consisting of parameterized computed probe characteristics along with the corresponding plasma, satellite and instrument conditions assumed in the simulations. Multivariate regression techniques are then applied to train and validate models using synthetic data from these solution libraries. Models trained over a given range of plasma parameters, can then be used to quickly infer physical parameters from characteristics. An added advantage of adopting a machine learning approach, whereby trained models are tested on validation sets distinct from the training sets, is that they naturally yield inference uncertainties, or confidence intervals; a product which is not available from analytic inference techniques. Yet another feature of the simulation approach, is that it is generally straightforward to add physical effects or to modify simulations, to adapt to a particular geometry and plasma environment.

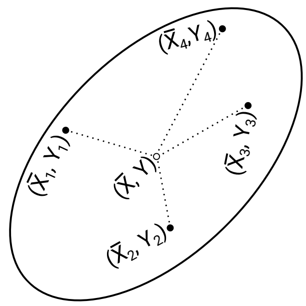
Direct solution: Simulations and construction of solution libraries

Three-dimensional kinetic simulations are made for space environment conditions representative of satellites in low Earth orbit (LEO), using the International Reference Ionosphere (IRI) model, assuming different times, latitudes, longitudes, and altitudes. Computed currents collected, with input parameters used in the simulations, are recorded in a solution library; that is, a synthetic data set to be used in supervised training.



Inverse problem: Regression-based model training and validation

Two regression techniques are considered. Radial Basis Function regression is the simplest one to implement. Given a training data set with independent variables (current collected by probes), parameterized with n-tuples $\bar{X}$ and dependent variables Y to be inferred (density, temperature, etc.), selected entries, or nodes of the set are used to “interpolate” dependent variables corresponding for arbitrary values of $\bar{X}$, from a linear superposition of a functions of the Euclidean distance, or L^2 norm between $\bar{X}$ and selected values $\bar{X}_i$ of nodes in the training set:



$$\tilde{Y} = \sum_{i=1}^{N} a_i G(|\bar{X} - \bar{X}_i|) Y_i,$$

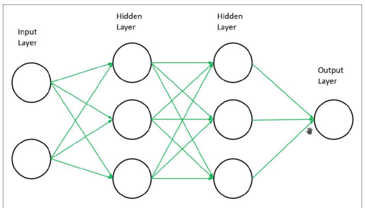
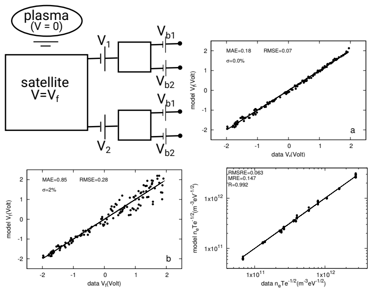


FIGURE 7 | Schematic of a feed forward neural network. Arrows represent the weight values that are optimized during the training of a neural network model and circles represent the nodes.

where a_i are interpolation coefficients, $\tilde{Y}$ is the interpolated, or inferred value, G is a suitable interpolating function, the selected $\bar{X}_i$ are interpolating centres, and Y_i are known dependent variables at the centres. The second technique considered uses deep learning neural networks. It is a little more complicated than RBF, but it is more general and powerful, especially when working with very large data sets.

Application 1: Two spherical probes at fixed bias voltages

In this example, we assess the possibility of inferring a satellite potential from a 2-tuple of currents collected by two spherical probes at fixed bias voltages. A training, and a distinct validation data set are constructed from kinetic simulations, as described above, to train and validate RBF inference models for the satellite floating potential V_f, and $n_e/\sqrt{T_{eV}}$, where T_{eV} is the electron temperature in eV. These satellite and plasma parameters are of interest because, in the orbital motion limited (OML) approximation, when currents I_1 and I_2 are measured with fixed voltages V_1 and V_2, it is possible to derive the following expressions:



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$$V_f + T_{eV} = \frac{V_{b1}I_2 - V_{b2}I_1}{I_1 - I_2},$$

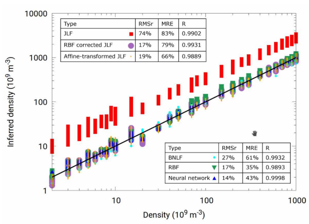
and

$$\frac{n_e}{T_{eV}} = \frac{1}{er^2}\sqrt{\frac{m_e}{8\pi e}}\left(\frac{I_1 - I_2}{V_{b2} - V_{b1}}\right),$$

where r is the radius of the assumed spherical Langmuir probe, and e is the elementary charge. These analytic expressions are found to produce good approximations for V_f and $n_e/\sqrt{T_{eV}}$ in the left hand sides. These approximations in turn can be improved by combining them with RBF regression. In this "boosting"[1] approach, RBF is effectively used to model the discrepancy between the analytic expressions and known values in the training set, and to apply a correction. The figures above show a hypothetical setup of such an instrument on a satellite, with correlation plots of the inferred floating potential without (top right), and with added noise (lower corner left) to the validation set. A correlation plot of inferred $n_e/\sqrt{T_{eV}}$ is shown in the lower right corner of the figure.

Application 2: NorSat-1 satellite fixed bias needle probes (mNLP)

The same technique is also applied to infer the electron density and the floating potential from 4-tuples of currents measured with four fixed biased needle probes. A solution library is first constructed from simulation results, consisting of 4-tuples of currents and plasma parameters assumed in the simulations, for randomly distributed values of a satellite potential, in the range −6



[1]With boosting two regression techniques are combined, such that one can be seen as correcting the other.

to -1 V. The solution library is then used to construct a training set consisting of randomly selected entries, and a validation set, consisting of the remaining entries in the library. Models are then trained and validated, first on the sole basis of these synthetic data sets. The models trained with synthetic data are then applied to actual currents measured by the four needle probes on the NorSat-1 satellite. The correlation plot for the inferred densities against known densities from the validation set are shown in the last figure, as obtained with different models. These are the Jacobsen linear fit (JLF)[2], JLF corrected with RBF, JLF corrected with a simple affine transformation, Barjatya's nonlinear fit (BNLF)[3], and neural network. Except for the original JLF, which overestimates densities by approximately a factor 3, all inferences are in good quantitative agreement with densities from the validation set, with RMS relative errors ranging from 14% to 27%. A point noteworthy in this last figure, is the fact that the skill of the JLF inferences can be increased significantly, by applying a simple affine transformation to the log of the inferred densities. This is particularly interesting, considering the simplicity of the JLF technique, and of an affine transformation, compared to the other inference techniques. For this reason, this simple combined approach, while not necessarily the most accurate, could very well be the preferred inference technique in practical data processing. Models trained with synthetic data are now being used to infer electron densities, and satellite potentials from in situ currents measured in situ, with NorSat-1 fixed bias needle probes. This work is currently in progress, and will be reported in a forthcoming publication.

Conclusion

A new approach, based on computer simulations and multivariate regression methods, has been presented with the goal of improving inferences of plasma and satellite physical parameters from currents collected by fixed bias Langmuir probes. The approach follows machine learning procedures, utilizing a solution library consisting of pre-computed currents for specified plasma conditions. This library is used to construct training and distinct validation sets with which regression inference models can be trained and assessed for their accuracy. The advantage of this approach, compared to traditional techniques based on theory and analytic approximations, is that it can account for more realistic physical processes and geometry. It naturally produces uncertainties in inferences, that are specific to the inference algorithm. It also enables incremental improvements to inference models, by including more physical processes and more detailed geometry, as required in a given problem.

[2]https://doi.org/10.1088/0957-0233/21/8/085902
[3]https://doi.org/10.1063/1.5022820