Crescent-shaped spot modeling of runaway electron synchrotron radiation

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Introduction: The energy of disruption generated runaway electrons (REs) can reach as high as tens of MeV and they can cause serious damage to plasma-facing surfaces in large tokamaks and ITER (see, e. g., [1]). At the same time, the quiescent RE generation in low density Ohmic discharges is used for a more detailed and accurate measurement of RE parameters (see, e.g., [2]). The investigation of RE synchrotron radiation spot allows studying the parameters: RE beam size and position, number, and maximum energy of REs. The parameter $\theta_{\mathrm{p}} = v_{\perp} \big/ \left| v_{||} \right|$ (pitch angle) strongly influences the RE synchrotron radiation behavior ($v_{||}$ is the longitudinal velocity, $v_{\perp}$ is the transverse velocity with respect to the magnetic field $\vec{B}$). Because in tokamak experiments the strong inequality $\theta_{\mathrm{p}} >> \gamma^{-1}$ takes place, the small uncertainty introduced by the angular broadening of radiation, $\sim \gamma^{-1}$, is possible to neglect (cone model), γ is the relativistic factor. The case of the small parameter $\theta_{\mathrm{p}} <<1$ (linear cone model) was considered in [3] and is used for experimental data analysis till now.

In the recent papers (see, e.g., [2,4]), a large pitch angle ($\theta_{\mathrm{p}} \sim 1$) was declared for the observed synchrotron emission (SE) crescent-shaped spots. And in [5] it was revealed that the dominating particle should have a $\theta_{\mathrm{p}} \sim 0.35$ rad, E=25 MeV (DIII-D, # 165826).

Nonlinear cone model equations: For the RE finite pitch angle values the analytical expressions for analysis of the SE in tokamaks have been obtained in [6] (nonlinear cone model, $\theta_{\mathrm{p}} \sim 1$), $\zeta = \pi/2 + \Delta$, see Fig. 1. They have the next form ($\dot{\alpha} \approx \omega_B$, $\omega_B(r,\theta) = eB(r,\theta)/\gamma m_e c$ $(e>0)$, D is the distance from the detector to YZ plane):

$$\pm\left[r\sin\theta \mp \frac{v_{\perp}}{\omega_B}\cos\left(\theta \pm \alpha\right) - Z_{\mathrm{det}} \right]\left[\cos\Delta \mp \frac{v_{\perp}}{v_{||}}\sin\Delta\cos\left(\theta \pm \alpha\right) \right] =$$
$$= \left[D + \left(R_0 - r\cos\theta\right)\sin\Delta \right]\left[\frac{r}{qR_0}\cos\theta + \frac{v_{\perp}}{v_{||}}\sin\left(\theta \pm \alpha\right) \mp \frac{1}{v_{||}}\frac{v_{||}^2 + 0.5v_{\perp}^2}{\omega_B R_0} \pm \frac{v_{\perp}}{v_{||}}\frac{v_{\perp}}{\omega_B r}\cos\left(\theta \pm \alpha\right)\cos\alpha \right], \quad (1)$$

$$\sin\Delta\left[1 \pm \frac{Y_{\mathrm{det}}}{D}\frac{v_{\perp}}{v_{||}}\cos\left(\theta \pm \alpha\right) \right] = \frac{Y_{\mathrm{det}}\cos\Delta - \left(R_0 - r\cos\theta\right)}{D} \mp \frac{v_{\perp}}{v_{||}}\cos\Delta\cos\left(\theta \pm \alpha\right), \quad (2)$$

The tokamak confinement magnetic field is presented in the form:

$$\vec{B}(r,\vartheta) = B_{0\zeta}R_0 \,/\, R\left(\pm\vec{e}_{\zeta} + r/q(r)R_0\vec{e}_{\theta} \right). \quad (3)$$

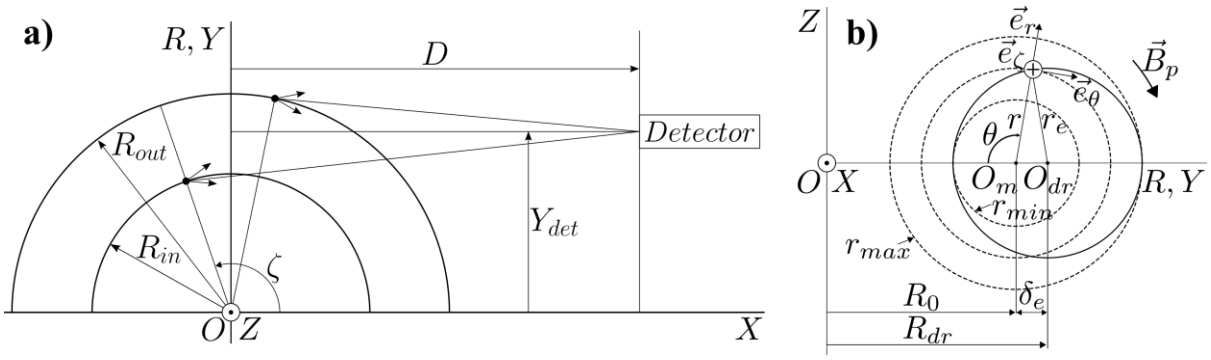

Figure 1. a) Relative positions of the runaway electrons beam and the detector are shown. For simplicity, the case of the plane $z = 0$ ($Z_{det} = 0$) is presented, R_{in} and R_{out} are the inner and outer major radii of the beam in this plane, the cones of radiation emitted by electrons are represented schematically; b) the cross-sections (through the plane $\zeta = \pi/2$) of the magnetic surfaces (dashed circles) with a center at the point O_m and the drift surface with shift δ_e (solid circle) at a center O_{dr} are plotted, r_{min} and r_{max} are minimal and maximal minor radii of magnetic surfaces corresponding to this drift surface with radius r_e. The X-axis is directed toward the detector. The direction of the poloidal magnetic field $\vec{B}_p$ is shown.

The upper signs in the Eqs. (1-3) correspond to the case when the magnetic field $\vec{B}$ is directed away from the detector, and for the lower signs field is directed toward the detector. For the RE moving toward the detector the longitudinal velocity $v_{||}$ is negative for the upper sign ($v_{||} < 0$) and positive for the lower sign ($v_{||} > 0$). The RE motion along the tokamak helical magnetic field with velocity $v_{||}$, cyclotron gyration motion around the guiding centre with the velocity $v_\perp$, safety factor *q(r)*, the horizontal displacement of the RE drift surfaces with respect to the magnetic surfaces, and the position of the detector are taken into account.

Note that besides the parameter $v_\perp / |v_{||}|$, the ratio Y_{det} / D is also the key parameter for the analysis of the synchrotron radiation spot shape and that for each poloidal angle θ the SE of REs is detected by the camera for certain values of α (cyclotron rotation angle) only.

Modeling of DIII-D experiment: The results of [6] are used for qualitative modeling of the DIII-D experiment [2], shot #152893, λ = 901nm where the SE crescent-shaped spot was observed (Fig. 5 in [2]) on the high-field side (HFS). Our calculations confirmed the result of [2]: when REs reached E = 30 MeV REs were no longer confined in the tokamak (see Fig. 2). The REs must have energy $E \gtrsim 25$ MeV to emit visible SE and be detected in the DIII-D experiment. That is why in Fig. 3 we present the SE spot modeling for RE energies 25 and 30 MeV. The SE is found in [2] to be absent from approximately inside the q = 1 surface and outside the q = 3 surface. In Fig. 2 the drift orbits are the solution of relativistic drift motion equations. In modeling, the starting points of REs drift orbits are on the HFS between q = 1 and q= 1.5 magnetic surfaces.

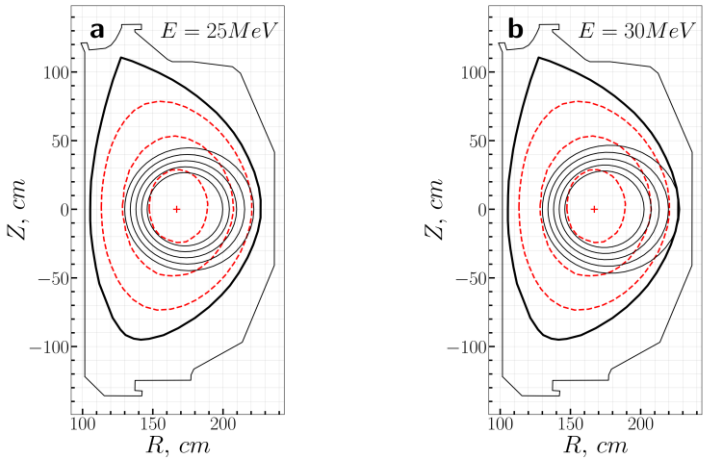

Figure 2. Modeling of runaway electron drift orbits for energies a) 25 MeV and b) 30 MeV.

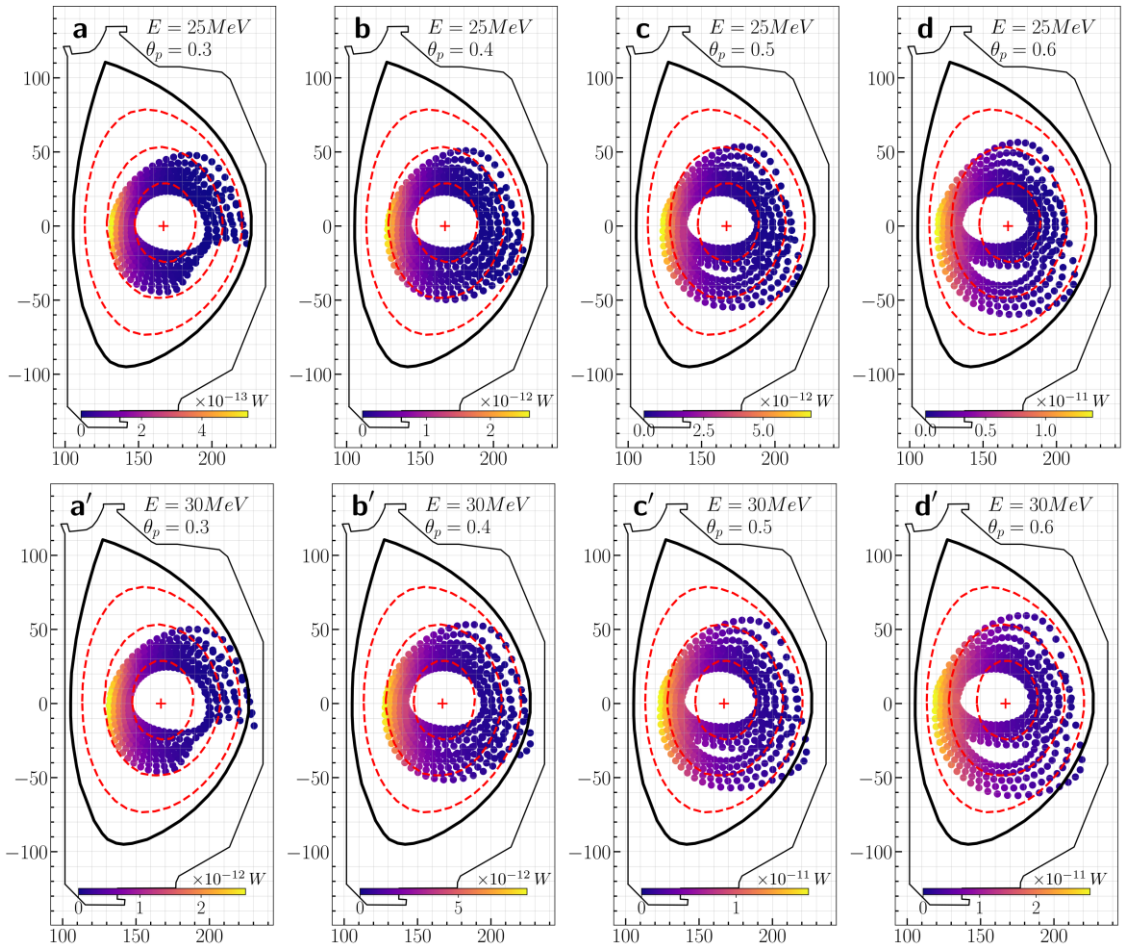

Figure 3. Modeling of synchrotron spots. For $\theta_p = 0.6$ the bright edge of SE spot is shifted to q = 3 like in Fig. 5(f) [2].

The magnetic surfaces q = 1, 1.5 and 3 are shown in Figs. 2,3 by red dashed lines. The values 0.3, 0.4, 0.5 and 0.6 of parameter θ_p were assigned at the q = 1.5 surface only and were varied along RE drift orbits according to the conservation of magnetic moment. The color of each

point in Fig. 3 corresponds to the value of $\int_{\lambda_{min}}^{\lambda_{max}} d\lambda P(\lambda)$, where ($\lambda_{min}$, λ_{max}) – detector's wavelength range and $P(\lambda) = i\frac{2\pi ce^2}{\lambda^3\gamma^2}\left\{\int_C \frac{du}{u}\left(1-2u^2\right)I_0\left(au^3\right)\exp\left[-\frac{3}{2}\xi\left(u-\frac{u^3}{3}\right)\right]-\right.$

$\left.-\frac{4\eta}{1+\eta^2}\int_C du\cdot uI_1\left(au^3\right)\exp\left[-\frac{3}{2}\xi\left(u-\frac{u^3}{3}\right)\right]\right\}$ is the full equation of spectral density [7], where $a=\frac{\xi\eta}{1+\eta^2}$, $\xi=\frac{4\pi R}{3\lambda\gamma^3\sqrt{1+\eta^2}}$, $\eta=\frac{v_\perp}{v_{dr}}$ and $v_{dr}=\frac{v_{||}^2+0.5v_\perp^2}{\omega_B R_0}$.

Conclusions: Presented results show that a crescent shape appears for a large pitch angle (θ_p ~ 1) and the relative absence of SE on the magnetic field LFS is an artifact of the conservation of magnetic momentum. Our calculations confirm the statements of Refs. 2,4,5.

A good agreement between the presented qualitative modeling and the DIII-D experiment [2] is obtained. In our opinion, the synchrotron radiation crescent-shaped spot shown in Fig. 5(f) corresponds to the value of parameter $v_\perp / |v_{||}| \approx 0.6$.

The values $v_\perp / |v_{||}|$ and Y_{det}/D are the key parameters for the analysis of the synchrotron radiation spot shape.

Acknowledgments. Authors thanks Z. Popovic for useful and fruitful discussions.

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