Energetic particle nonlinear equilibria and transport processes in burning plasmas

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Understanding the dynamics of burning plasma over long time scales, such as the energy confinement time or even longer, is a crucial issue. Most of the studies on core plasma transport are based on a systematic scale separation between the reference equilibrium and fluctuations. However, in fusion devices energetic particle (EP) transport is a spatiotemporal multi-scale phenomenon [1]. Even in drift wave plasma turbulence theory [2] and simulations [3,4], spatial-temporal mesoscales are recognized as important players. In a recent paper [5], we stressed the necessity of a self-consistent theory, which includes the dynamic determination of the reference state's characteristic spatiotemporal scales. In the following pages, we will briefly recall the concept of Phase Space Zonal Structures (PSZS), introduced as the phase space counterpart of macroscopic plasma equilibrium, and the Dyson Schrodinger transport model [6] that describes the coupled evolution of fluctuations spectral intensity and PSZS. Finally, we calculate the PSZS fluxes induced by a TAE on the Divertor Tokamak Test (DTT) facility [11]

Nonlinear equilibria

The study of fluctuation-induced transport in burning fusion plasmas raises a number of challenges; particularly for describing resonant EP transport in phase space. Resonant behaviors and collisionless EP transport create structures in phase space that can depart considerably from equilibrium in the absence of fluctuations; affecting, thus, transport on long time scales. In the absence of symmetry breaking fluctuations and sources/collisions, these PSZS are undamped by collisionless processes and do not evolve in time. Therefore, PSZS must be defined as functions of the adiabatic invariants of motion in the given reference "equilibrium," which may evolve in time [1,6,7]. The macro-/meso-scopic component of averaged over the unperturbed orbit distribution function is associated with the PSZS equation; which, following Ref. [1,5-8], can be written as:

$$\partial_t \left(\overline{e^{iQ_z}\bar{F}_0} \quad + \overline{e^{iQ_z}\delta F_z} \right) = -\frac{1}{\tau_b}\frac{\partial}{\partial \psi}\left[\tau_b \overline{e^{iQ_z}\delta\dot{\psi}\delta F}\right]_z - \frac{1}{\tau_b}\frac{\partial}{\partial \mathcal{E}}\left[\tau_b \overline{e^{iQ_z}\delta\dot{\mathcal{E}}\delta F}\right]_z$$
$$+ \overline{\left(\sum_b C_b^g [F, F_b] + \mathcal{S}\right)_{zS}} \tag{1}$$

Here, on the LHS, we have separated slow and fast spatiotemporal responses and we have introduced the shift operator e^{iQ_z}, describing the guiding center radial displacement due to equilibrium magnetic drifts, the bounce/transit time τ_b, and the collision and source terms. $\overline{(\ldots)} \equiv \tau_b^{-1} \oint (\ldots) d\theta/\dot{\theta}$ corresponds to bounce/transit averaging. At each instant, we can decompose the toroidally symmetric distribution function as follows:

$$F_z = \bar{F}_0 + \overline{\delta F_z} + \delta\tilde{F}_z \tag{2}$$

where the micro-scales are accounted for by $\overline{\delta F_z}$ while PSZS macro- & meso-scales are described by the first term on the l.h.s. in Eq. (1). PSZS together with the long-lived component of the electromagnetic fields, i.e., the zonal field structures (ZFS), define a state of nonlinear equilibrium in the presence of a finite level of symmetry breaking fluctuations; that is, the Zonal State (ZS). Assume, for simplicity, that ZS is characterized predominantly by the scalar potential $\delta\phi_Z$. We can study its self-consistent evolution in the absence of symmetry breaking fluctuations. The scalar ZFS is determined by the axisymmetric component of the quasi-neutrality condition:

$$\sum_s \left\langle \frac{e^2}{m}\frac{\partial F_0}{\partial \mathcal{E}} \right\rangle_v \delta\phi_z + \sum_s \left\langle e\hat{J}_0 \delta G_z \right\rangle_v = 0 \tag{3}$$

where:

$$G_z = e^{-iQ_z} G_{Bz} \equiv F_z - \frac{e}{m}\langle \delta L_g \rangle_z \left.\frac{\partial}{\partial \mathcal{E}}\right|_{\bar{\psi}} \bar{F}_0 + \frac{F(\psi)}{B_0}\langle \delta A_{\parallel g}\rangle_z \frac{\partial}{\partial \bar{\psi}} \bar{F}_0 \tag{5}$$

and

$$(\partial_t + \omega_b \partial_{\vartheta c})\delta G_B = -e^{-iQ_z}\left[\frac{e}{m}\frac{\partial F_{z0}}{\partial \mathcal{E}}\hat{J}_0 \partial_t \delta\phi_z + NL\right] \tag{4}$$

Substituting the orbit averaged component of Eq. (4) into Eq. (3) we obtain:

$$\sum_s \frac{V'_\psi}{4\pi^2}\frac{e^2}{m_s^2}\left\langle\left\langle \frac{\partial F_0}{\partial \mathcal{E}}\delta\phi_z - \overline{e^{-iQ_z}\hat{I}_0 e^{iQ_z}\hat{I}_0\frac{\partial F_{z0}}{\partial\mathcal{E}}\delta\phi_z}\right\rangle_v\right\rangle_\psi = \sum_s\sum_\sigma\int d\mathcal{E}d\mu\tau_B e\frac{i}{\omega}\overline{e^{iQ_z}\overline{\overline{\hat{I}_0 e^{iQ_z}NL}}} \qquad (6)$$

PSZS fluxes

In a recent work [6], we have introduced the so-called Dyson-Schrödinger transport model (DSM) describing the self-consistent evolution of the amplitude of electromagnetic fluctuations and the PSZS, giving a reduced description of EP transport and plasma dynamics. DSM can recover either the full nonlinear gyrokinetic description or the quasilinear approximation in the proper limits. We recall that the main difference with respect to the usual wave kinetic formalism is that wave amplitude is described by a nonlinear Schrödinger-like equation. Therefore, wave-packet radial propagation as well as focusing/defocusing due to radial nonuniformities and nonlinearities are treated on the same footing. By using this approach, consistently with the PSZS theoretical framework, the dimensionality of the problem is effectively reduced; resulting in a conceptual and computational simplification. Focusing on the excitation of Drift Alfvén Waves in burning plasmas, it is possible to show that governing equations in the reduced model can be calculated up to the required accuracy by means weighing over linear parallel mode structures.

Following this approach, we take as an example the parallel mode structure of the scalar and vector potentials of an n=20 TAE excited by EP in the DTT (cf. Fig. 2). For the sake of simplicity, we have considered only circulating particles. Realistic magnetic geometry has been described by using the EQUIPE equilibrium post processor of FALCON [10].

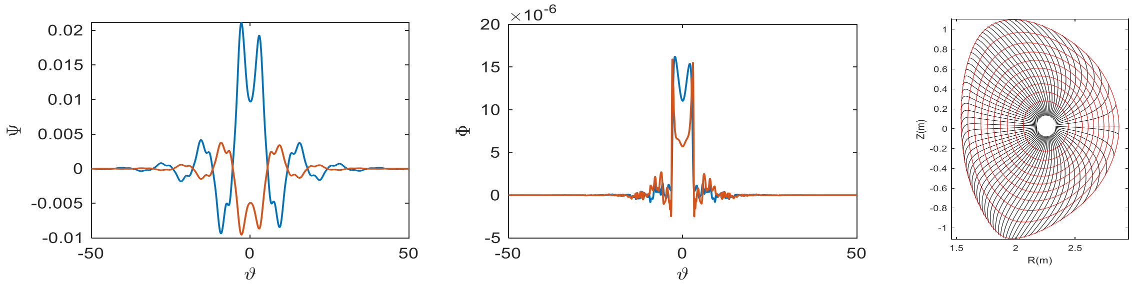

Figure 1: parallel mode structures of $\Psi = (e/T)(\omega/k_\parallel c)\delta A_\parallel$ and the scalar potential Φ adopting Boozer coordinates for DTT full power scenario [9]

Nonlinear Phase space particle fluxes can be calculated by direct substitution in the PSZS governing equation. Note that the resonant contribution is dominant, as expected by the theory.

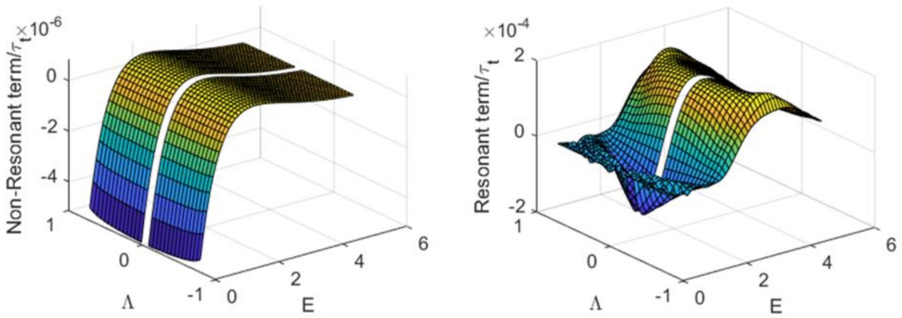

Figure 2: TAE induced diffusive flux calculated by DAEPS on DTT (Divertor Tokamak Test)] due respectively to non-resonant and resonant particles

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