Quasilinear Gyrokinetic Modeling of Reduced Transport in the Presence of High Impurity Content, Large Gradients, and Large Geometric α_{MHD}

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Introduction

Transport barriers in tokamak discharges are often characterized by large gradients that can destabilize electrostatic microinstabilities, thereby driving anomalous turbulent transport [1]. However, large gradients can also lead to large geometric α_{MHD}, a stabilizing parameter in certain regimes [2]. The resulting transport is inherently constrained to be ambipolar; in effect, these large gradients can make this flux constraint impossible to satisfy, resulting in stabilization and the reduction of turbulent transport [3]. Due to the high computational cost of nonlinear gyrokinetic simulations, using a reduced turbulent transport model is ideal for predictive modeling. However, reduced models tailored for the tokamak core can become unreliable in transport barrier regimes, thus necessitating model development and improvement. We test the extent to which the gyrokinetic quasilinear code QuaLiKiz [4] can reliably predict anomalous transport in transport barrier discharge regimes to determine parameters that lead to turbulent transport reduction. We use the gyrokinetic code GENE [5], based on first principles, as a point of comparison for QuaLiKiz. Unlike GENE, QuaLiKiz uses many approximations to ensure computational tractability. In particular, QuaLiKiz assumes a Gaussian eigenfunction, uses $s-\alpha_{\mathrm{MHD}}$ geometry, and only captures electrostatic fluctuations. To ensure accurate predictions in transport barrier discharge scenarios, we improve the approximations made for trapped particles, and thus the trapped electron mode (TEM), by incorporating the bounce-averaged electrostatic eigenfunction [6, 7]. The Gaussian ansatz allows us to analytically estimate this bounce-averaging effect with sufficient accuracy. We also seek to improve the mode structure approximation in order to accurately calculate bounce-averaging effects.

The Gyrokinetic Model QuaLiKiz

QuaLiKiz is a quasilinear gyrokinetic code that computes the turbulent particle, angular momentum, and heat transport using the quasilinear approximation. The code only models electrostatic microinstabilities in $s-\alpha_{\text{MHD}}$ geometry; furthermore, it assumes that the potential eigenfunction $\hat{\phi}$ is strongly ballooned. The dispersion relation is

$$D(\omega) = \sum_s \mathrm{d}^3x\,\mathrm{d}^3v\,\frac{e_s^2 f_{0,s}}{T_s}\left(1 - J_0^2\frac{\omega - n\omega_*}{\omega - k_\parallel v_\parallel - n\omega_d}\right)|\phi|^2 . \tag{1}$$

We then obtain a 0-D eigenvalue equation for the complex frequency ω. Note that the electrostatic potential ϕ is not solved self-consistently; instead, we derive an approximate form for the ballooning structure using a high-frequency expansion procedure. Once we determine the growth rate and real frequency, the turbulent fluxes are calculated using a saturation rule.

To determine QuaLiKiz's accuracy in the investigated parameter regime, we benchmark the code with the first-principles-based gyrokinetic code GENE. Linear initial value simulations of GENE determine the growth rates, which can then be compared to QuaLiKiz predictions. Meanwhile, we use nonlinear simulations of GENE to compare turbulent fluxes. After first comparing turbulent flux predictions between QuaLiKiz and GENE by scanning over large gradients and α_{MHD} consistently, we find that QuaLiKiz is incorrect by an order of magnitude for high α_{MHD} (not shown here). In order to diagnose the discrepancy, we perform a focused analysis of the linear physics.

High α_{MHD} Drift Reversal and Bounce Averaging

In QuaLiKiz, the regime-relevant term that most directly depends on α_{MHD} is the bounce-averaged drift frequency in the trapped electron response. The normalized bounce-averaged drift frequency is given by

$$F_d(\kappa) = -1 + \frac{2E(\kappa)}{K(\kappa)} + 4\hat{s}\left(\kappa^2 - 1 + \frac{E(\kappa)}{K(\kappa)}\right) - \frac{4}{3}\alpha_{\text{MHD}}\left(1 - \kappa^2 - \left(1 - 2\kappa^2\right)\frac{E(\kappa)}{K(\kappa)}\right). \tag{2}$$

Here, K and E are respectively the complete elliptic integrals of the first and second kind, and κ is a pitch angle parameter defined as $\kappa = \sin(\theta_b/2)$, where θ_b is the poloidal bounce angle. Thus, $\kappa = 0$ corresponds to deeply trapped particles and $\kappa = 1$ corresponds to barely trapped particles. We plot F_d against κ for different values of α_{MHD} with $\hat{s} = 1$. For large values of α_{MHD}, we observe a drift reversal where the drift frequency changes sign. This provides a stabilizing effect for the trapped electron mode; it is clear that for larger values of α_{MHD}, a greater proportion of velocity space will then stabilize the trapped electron mode.

We observe this stabilizing effect in both QuaLiKiz and GENE simulations by scanning over α_{MHD} in linear scans. However, we observe that the stabilizing response in QuaLiKiz is far greater. After some deeper analytical investigations, we discovered an error in the trapped electron portion of the dispersion relation. In the current version of the code, the trapped electron response is weighted by an average over the Gaussian eigenfunction squared that is performed separately. However, the correct treatment requires weighting the response against the square of the *bounce-averaged* eigenfunction. Since the bounce-average operator converts a poloidally dependent function to a pitch-angle dependent function, this results in a non-trivial weighting. Given a a Gaussian eigenfunction written as

$$\hat{\phi} \sim e^{-\theta^2 w^2/2d^2}, \tag{3}$$

where w and d are constant parameters that determine the mode width, the bounce average is

$$\left\langle e^{-\theta^2 w^2/2d^2} \right\rangle_b = \frac{1}{K(\kappa)} \int_0^1 \frac{\mathrm{d}u\, e^{-2(\mathrm{asin}(\kappa u))^2 w^2/d^2}}{\sqrt{1-\kappa^2 u^2}\sqrt{1-u^2}}. \tag{4}$$

By performing a Taylor expansion for small κ, we find that

$$\begin{aligned}\left\langle e^{-\theta^2 w^2/2d^2} \right\rangle_b \approx \frac{\pi}{2K(\kappa)} \Bigg(&\Gamma_0\left(\frac{\kappa^2 w^2}{d^2}\right)\left(1+\frac{\kappa^2}{4}+\frac{3\kappa^4}{16}-\frac{\kappa^4 w^2}{3d^2}\right) \\ &-\kappa^2 \Gamma_1\left(\frac{\kappa^2 w^2}{d^2}\right)\left(\frac{1}{12}+\frac{3d^2}{32w^2}+\frac{3\kappa^2}{16}-\frac{\kappa^2 w^2}{3d^2}\right)\Bigg)\end{aligned} \tag{5}$$

Here, $\Gamma_n(x) = \exp(-x) I_n(x)$. We plot the above function for various values of w/d and find that it strongly peaks at $\kappa = 0$ and decays as $\kappa \longrightarrow 1$.

For regime-relevant values of w/d, the bounce-averaged eigenfunction monotonically decreases with κ. Therefore, barely trapped particles must contribute a proportionally weaker response to the mode. This explains the overdamped QuaLiKiz results: drift reversal only occurs for relatively large values of κ. To produce the correct result, this stabilization effect must be appropriately weighted and therefore weaker than naively expected.

Simulations and Results

To validate the corrections to QuaLiKiz, we compare growth rates obtained from linear simulations. Our first comparisons use modified GA-Standard parameters (with smaller R/L_{T_i} to ensure the TEM dominates) with $k_\theta \rho_s = 0.2$. The scans performed are collisionless in order to isolate α_{MHD} and gradient effects; we also set GENE to use electrostatic and $s-\alpha_{\mathrm{MHD}}$ geometry settings to perform a like-for-like comparison. We report significant improvement after introducing the bounce-average effect in QuaLiKiz, even with $\alpha_{\mathrm{MHD}} = 0$, using this parameter set GA-Standard.

After performing other linear scans over other parameters (such as over the collision frequency or the magnetic shear), however, we large discrepancies for high values of the safety factor q. The bounce-average effect strongly depends on the width of the Gaussian eigenfunction. This width is calculated using a high-frequency fluid approximation; we have found that this estimate changes

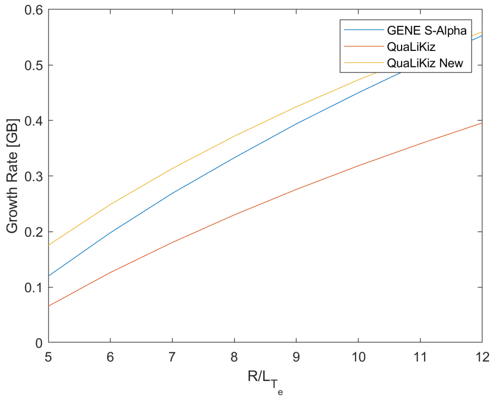

Figure 1: TEM growth rates against R/L_{T_e} with $\alpha_{\text{MHD}} = 0$; note we find improvement even when $\alpha_{\text{MHD}} = 0$.

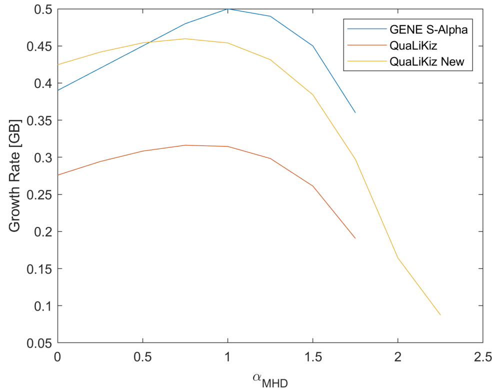

Figure 2: TEM growth rates against α_{MHD} at $R/L_{T_e} = 9.0$; we find improvement in both the small and large α_{MHD} regime.

strongly and monotonically with the safety factor (all else held fixed), thus overstabilizing the mode. This is unphysical, as seen from the GENE simulation results as well as examining the ballooning mode structure for different values of q. The next step in refining this correction is to produce more accurate ballooning mode structures so that the approximation remains valid at high q.

Figure 3: TEM growth rates plotted against safety factor q for $R/L_{T_e} = 9.0$, $\alpha_{\text{MHD}} = 0$.

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