Influence of electric potential on electrostatic microinstability in advanced stellarator

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Introduction

Turbulent transport is a crucial issue in the study of plasma confinement in stellarators. Recent experimental and simulation findings have revealed that the electric potential can exert an influence on the turbulence properties[1,2]. Then an important question arises: How does the electric potential affect the properties of microinstabilities and the resulting turbulence, such as the ion temperature gradient (ITG), trapped electron mode (TEM) etc[3-5]. This work focused on analyzing the effects of electric potential on electrostatic microinstabilities (ETG modes) in advanced stellarators using gyrokinetic theory, which retains the finite Larmor radius effect[6].

Theoretical Analyses

In this study, we investigate a magnetic configuration with nested magnetic flux surfaces in stellarators, which enable us to express the magnetic field using toroidal magnetic flux ψ and Clebsch angle α as $\boldsymbol{B} = \boldsymbol{B_0} = \nabla\psi \times \nabla\alpha$. Our analysis assumes that the equilibrium electric field is of order $e\phi_0(\psi)/T \sim O(0)$ and the standard gyrokinetic ordering $\frac{\omega}{\Omega} \sim \frac{k_\parallel}{k_\perp} \sim \frac{e\tilde{\phi}}{T} \sim \epsilon \ll 1$. Consequently, we can drive the linearized Vlasov equation in electrostatic approximation for species $a = e$ and i as follows:

$$\left[\frac{\partial}{\partial t} + \boldsymbol{v} \cdot \frac{\partial}{\partial \boldsymbol{r}} + \frac{q_a}{m_a}(\boldsymbol{v} \times \boldsymbol{B_0} - \nabla\phi_0) \cdot \frac{\partial}{\partial \boldsymbol{v}}\right] f_{a1} = \frac{q_a}{m_a} \nabla\tilde{\phi} \cdot \frac{\partial}{\partial \boldsymbol{v}} f_{a0}, \quad (1)$$

where f_{a0} denotes the equilibrium distribution function

$$f_{a0} = \frac{n_a(\psi)}{(2\pi T(\psi)/m_a)^{3/2}} e^{-(\varepsilon - q_a\phi_0(\psi))/T(\psi)},$$

with n_a being particle density, and $f_{a1} = g_a - \frac{q_a\tilde{\phi}}{T_a} f_{a0}$ the perturbed distribution function with g_a being non-adiabatic component. After many tedious manipulations, we obtain the linearized gyrokinetic equation that determines g_a,

$$iv_\parallel \nabla_\parallel g_{a0}(\boldsymbol{k}_\perp, \mu, \varepsilon, \omega, l) + (\omega - \omega_{da}) g_{a0}(\boldsymbol{k}_\perp, \mu, \varepsilon, \omega, l) = \frac{q_a}{T_a} \tilde{\phi}(\boldsymbol{k}_\perp, \omega, l) J_0(k_\perp \rho) \left(\omega - \omega_{*a}^T - k_\alpha \frac{\partial \phi_0}{\partial \psi}\right) f_{a0}, \quad (2)$$

where $\omega_{*a}^T = \omega_{*a}[1 + \eta_a(m_a v^2/2T_a - 3/2)]$ with $\eta_a = dlnT_a/dlnn_a$ and $\omega_{*a} = (k_\alpha T_a/q_a)\mathrm{dln}n_a/d\psi$ the diamagnetic frequency, and $\omega_{da} = \boldsymbol{k}_\perp \cdot \boldsymbol{v}_{da}$ denotes the drift frequency with $\boldsymbol{v}_{da} = \left(v_\parallel^2 + \frac{v_\perp^2}{2}\right) \hat{\boldsymbol{b}} \times \nabla \mathrm{ln} B_0/\Omega_a + \boldsymbol{E_0} \times \boldsymbol{B_0}/B_0^2$ the magnetic plus electric drifts. The system of equations is closed by incorporating Poisson's equation. For simplified case of $T_i = T_e = T$, Poisson's equation can be written as

$$\left(k_\perp^2 \lambda_{De}^2 + 2 + \frac{\varepsilon_0}{en_e} \nabla \cdot \boldsymbol{E_0}\right) \tilde{\phi}(\boldsymbol{k}, \omega) = \frac{\mathrm{T}}{en_e} \int (g_i(\boldsymbol{k}, \mu, \varepsilon, \omega) J_0(k_\perp \rho_i) - g_e(\boldsymbol{k}, \mu, \varepsilon, \omega) J_0(k_\perp \rho_e)) d\vec{v}, \quad (3)$$

here, $\lambda_{De}^2 = \varepsilon_0 T/e^2 n_e$ denotes the Debye length of electron. Then, a dispersion equation without additional assumptions can be obtained by substituting the expressions for g_i and g_e into this function.

By utilizing $\nabla_\parallel = ik_\parallel$ and considering the low-frequency limit, $\omega \ll k_\parallel v_T$, in Eq. (2), we derive

$$g_a = \frac{\omega - \omega_{*a}^T - k_\alpha \frac{\partial \phi_0}{\partial \psi}}{\omega - \overline{\omega}_{da}} \frac{\overline{q_a J_0(k_\perp \rho_a) \tilde{\phi}(\boldsymbol{k}, \omega)}}{T_a} f_{a0}, \quad (4)$$

where the overhead bar denotes an orbit average. And the bounce average of the drift frequency $\overline{\omega}_{da}$ is associated with the adiabatic invariant J and can be expressed as $\overline{\omega}_{da} = \frac{k_\psi}{q_a \tau_b} \frac{\partial J}{\partial \alpha} - \frac{k_\alpha}{q_a \tau_b} \frac{\partial J}{\partial \psi}$ with $\tau_b = \int \frac{dl}{v_\parallel}$ representing the bounce time. In this context, the passing particles are disregarded due to their significantly smaller response coefficient, $\omega/k_\parallel v_T \ll 1$, compared to that of trapped particles. Substituting this equation into Eq. (3) and considering adiabat ion and kinetic electron, we derive an integral function

$$\left(k_\perp^2 \lambda_{De}^2 + 2 + \frac{\varepsilon_0}{en_{e0}} \nabla \cdot \boldsymbol{E_0}\right) \tilde{\phi}(\boldsymbol{k}_\perp, \omega, l) = \frac{1}{n_e} \int \frac{\omega - \omega_{*e}^T - k_\alpha \frac{\partial \phi_0}{\partial \psi}}{\omega - \overline{\omega}_{de}} f_{e0} \overline{J_0(k_\perp \rho_e) \tilde{\phi}(\boldsymbol{k}_\perp, \omega, l)} J_0(k_\perp \rho_e) d\vec{v}. \quad (5)$$

Multiplying this equation by $J_0 \tilde{\phi}^*/B$ and integrating over velocity space and along the magnetic field yields