Stability analysis of the axi-symmetric vertical mode in MHD simulation

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The characteristics of axi-symmetric modes, toroidal mode n=0, in tokamak is crucial to know in perspective of a steady continuous operation of plasma. It is known that this mode either becomes unstable in experiments, leading to vertical displacement events, or oscillatory as observed in a few experiments on JET. A series of analytical calculation [1, 2] have been done recently based on a model straight tokamak configuration to gain a better understanding of these modes' properties. In this article, we report the numerical verification of those analytic results. For an elongated plasma in straight tokamak, the analytical calculation unfolds three stability conditions of the n=0 mode - unstable, marginally stable and oscillatory - with varying location of an ideal wall.

The linear numerical simulation has been carried out with the initial value code NIMROD that solves the extended MHD equations using implicit/semi-implicit time-advance algorithm based on finite element method [3]. The following MHD equations are solved in our simulation:

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\mathbf{v}) = \nabla\cdot D\nabla n \tag{1}$$

$$mn\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right)\mathbf{v} = \frac{1}{\mu_0}(\nabla\times\mathbf{B})\times\mathbf{B} - \nabla p + \nabla\cdot\nu\rho\nabla\mathbf{v} \tag{2}$$

$$\frac{3}{2}\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right)T = -nT\nabla\cdot\mathbf{v} \tag{3}$$

$$\frac{\partial\mathbf{B}}{\partial t} = -\nabla\times\left[\eta_e(\nabla\times\mathbf{B}) - \mathbf{v}\times\mathbf{B}\right] + \kappa_{divb}\nabla\nabla\cdot\mathbf{B} \tag{4}$$

where $\mathbf{v}$ is the center-of-mass velocity of plasma with particle density n and ion mass m, p is the combined pressure of electron and ion, T represents the both ion and electron temperature; the magnetic field is denoted by B and the $\nabla\cdot\mathbf{B} = 0$ condition is enforced via a cleaning operator ($\kappa_{divb}\nabla\nabla\cdot\mathbf{B}$). The kinetic viscosity (ν) in the momentum equation and the number diffusivity (D) in the density equation are employed to suppress any numerical error due to sharp gradients in the plasma profiles. To setup equilibrium the following force balance relation for a straight tokamak plasma column is considered,

$$\frac{d}{d\psi}\left[\frac{B_z^2}{2\mu_0} + p(\psi)\right] = -J_z(\psi) \tag{5}$$

In conformity with the analytic theory [4], the current density $J_z(\psi)$ is modeled as uniform, a finite value (J_0) within the elliptical plasma ($\psi \leq \psi_b$) and zero outside, where ψ_b defines the value of poloidal flux at the elliptical plasma boundary. The Ampere's equation ($\nabla^2\psi = \mu_0 J_z(\psi)$) was solved analytically to obtain the poloidal flux function and the poloidal magnetic field as discussed in Ref. [4]. For simplicity, we have assumed a finite constant value of the axial magnetic field across the whole simulation domain. Therefore, Eq. 5 may be solved to obtain the pressure profile in ψ as $p(\psi) = p_0(1-\frac{\psi}{\psi_b}) = J_0\psi_b(1-\frac{\psi}{\psi_b})$ by imposing zero pressure at and outside the elliptical plasma boundary.

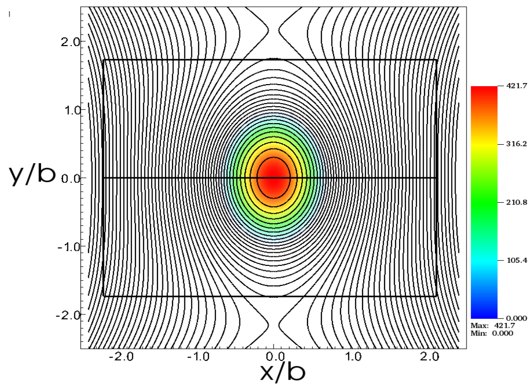


Figure 1: *The equilibrium plasma pressure is shown as colour-mapped onto the poloidal flux contours. The two X-points are visible above and below the elliptical plasma. The boundary of simulation domain are shown by the black rectangular box.*

The constant value of axial magnetic field is determined from the relation $B_z = B_0 = \sqrt{2\mu_0 p_0/\beta_0}$ for a given plasma axial beta (β_0) at the magnetic axis, which is defined as the ratio of the plasma pressure on axis to the axial magnetic pressure. Plasma density is modelled as a hyperbolic tangent function centered at the plasma boundary as $n(\psi) = n_h + (n_o - n_h)[1-\tanh[\alpha(\psi-\psi_b)/\psi_b]]/2$, where n_h and n_o are the values of density in the halo region and inside the elliptic plasma respectively. The obtained profiles of equilibrium plasma pressure, density and magnetic fields are mapped onto a straight slab geometry in Cartesian co-ordinates in NIMROD where the z-axis along the length of the slab represents the axi-symmetric direction of a straight tokamak, and the x-y plane the poloidal cross-section of it.

When the perfectly conducting boundary includes the X-points inside the simulation domain the n=0 mode becomes unstable, showing the signature of a vertical displacement event (VDE). This is evident in the vector plot of perturbed poloidal velocity of the n=0 mode in Fig. 2 as the flow vectors point upwards within the elliptical plasma and downwards outside. As the boundary wall is brought closer to plasma leaving the X-points outside of the do-

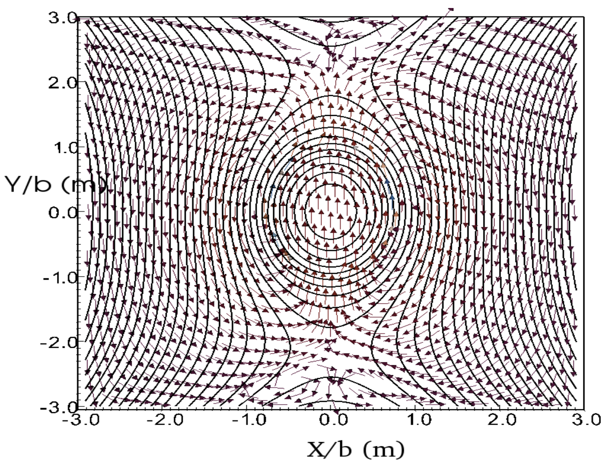
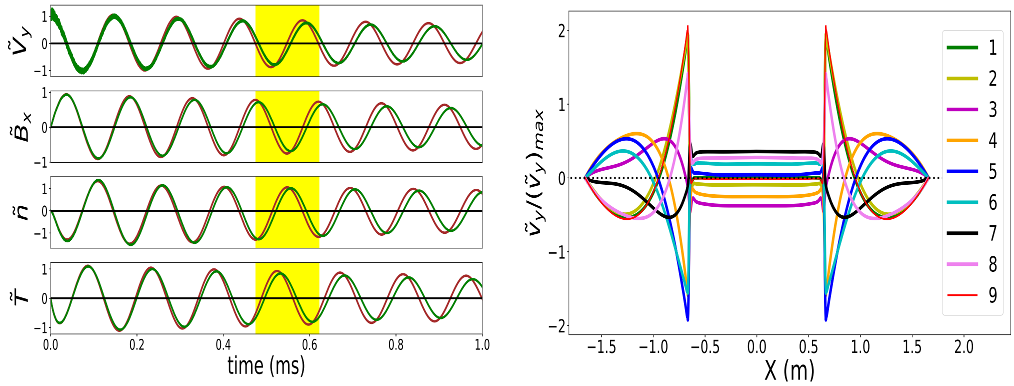


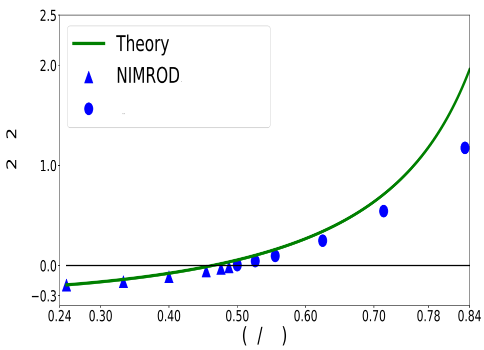
Figure 2: *The perturbed poloidal flow vectors within the elliptical plasma boundary indicate the upward movement of plasma.*

main the mode becomes oscillatory. The time history of the oscillating fields are shown in Fig. 3 with a demonstration of rigid-body oscillation, clearly visible in the 1D profiles. The green line



(a) time history of perturbed fields (b) 1D demonstration of rigid oscillation

Figure 3: a) Time history of oscillatory perturbed fields $\tilde{v}_y, \tilde{B}_x, \tilde{n}, \tilde{T}$ normalized by the respective amplitude of each field from a run with $\kappa = 1.4$ and $b/b_w = 0.55$. b) Perturbed $\tilde{v}_y/(\tilde{v}_y)_{max}$ along the horizontal black line in the mid plane (see Fig. 1) are shown at nine consecutive times at even interval within a cycle. The time interval of this cycle is marked in Fig. 3a by a yellow patch. These 1D profiles should be followed sequentially from 1 to 9 to get the picture of a full oscillation.



(a) wall position scan for $\kappa = 1.4$ (b) plasma elongation scan for $b/b_w = 0.55$

Figure 4: Comparison of normalised growth rate and frequency between theory and simulation.

curves in Fig. 4 demonstrates all the three stability conditions from the analytic theory; the segments below and above zero indicate respectively the growing and oscillatory phases while the zero cross-point denotes marginal stability for the scenario with ideal wall passing through the X-points. The blue circles (oscillatory cases) and triangles (unstable cases) in Fig. 4 from the NIMROD results are in close proximity of the theoretical curve, showing a good agreement between theory and simulation [5].