Energy dissipation in microtearing turbulence

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Introduction

Prediction of the thermal transport level due to microtearing (MT) turbulence, driven by electron temperature gradients, is a crucial issue for designing future fusion devices since MT turbulence leads to electron anomalous thermal transport. The thermal transport coefficient χ_e is related to the saturation amplitude of magnetic fluctuations $|\tilde{B}_x|$ as $\chi_\mathrm{e} \sim |\tilde{B}_x|^2/B_{z0}^2$ [1], where B_{z0} is the background magnetic field. To predict the transport level, the saturation level of magnetic fluctuations is required. According to a model developed by Ref. [2], one expects $|\tilde{B}_x|/B_{z0} \sim \rho_\mathrm{e}/L_{T_{0\mathrm{e}}}$, which is derived in the collisional limit, $\nu_\mathrm{ei}/\omega \gg 1$, of a drift-kinetic model, neglecting parallel dynamics, where ν_ei and ω are the electron-ion collision frequency and the interest mode frequency, respectively, and ρ_e is the electron Larmor radius and $L_{T_{0\mathrm{e}}}$ is the electron temperature gradient length.

Recent work [3] has demonstrated that the collisional MT instability survives in the weakly collisional regime $\nu_\mathrm{ei}/\omega < 1$. In weakly collisional plasmas, the phase mixing due to kinetic effects, such as finite Larmor radius (FLR) effects and Landau damping, creates a fine-scale structure in the velocity space, which leads to energy dissipation by collisions. Estimation of $|\tilde{B}_x|/B_{z0}$ using the drift-kinetic model in [2] does not include the nonlinear phase mixing effect. Furthermore, linear phase mixing is not also considered because of ignoring parallel dynamics. The effect of phase mixing on the saturation level of magnetic fluctuations is still obscure.

We perform nonlinear gyrokinetic simulations of the MT mode in a 2-dimensional slab domain using the electromagnetic gyrokinetic simulation code `AstroGK` [4] and investigate the contribution of the phase mixing to the saturation level of the magnetic fluctuations.

Phase mixing effect in the saturation amplitude level of magnetic field fluctuations

We consider an inhomogeneous plasma with density and temperature gradient embedded in a sheared magnetic field given by $\boldsymbol{B}_0 = B_{z0}\hat{\boldsymbol{z}} + B_y^\mathrm{eq}(x)\hat{\boldsymbol{y}}$ ($B_{z0} \gg B_y^\mathrm{eq}$). In a 2-dimensional slab, the electron gyrokinetic equation is described in the Fourier space as

$$\begin{aligned}\frac{\partial h_{\mathrm{e},k_y}}{\partial t} &+ \mathrm{i}k_\parallel v_\parallel J_0 h_{\mathrm{e},k_y} + \mathscr{F}\left(\{\langle\chi\rangle_{\boldsymbol{R}_{\mathrm{e},k_y}}, h_{\mathrm{e},k_y}\}\right) \\ &= -\mathrm{i}k_y \frac{f_{0\mathrm{e}}}{B_{z0}}\langle\chi\rangle_{\boldsymbol{R}_{\mathrm{e},k_y}}\left(L_{n_{0\mathrm{e}}}^{-1} + \left(\frac{v^2}{v_\mathrm{th,e}^2} - \frac{3}{2}\right)L_{T_{0\mathrm{e}}}^{-1}\right) - \frac{ef_{0\mathrm{e}}}{T_{0\mathrm{e}}}\frac{\partial}{\partial t}\langle\chi\rangle_{\boldsymbol{R}_{\mathrm{e},k_y}} + C(h_{\mathrm{e},k_y}), \qquad (1)\end{aligned}$$

where $\langle\chi\rangle_{\boldsymbol{R}_{\mathrm{e},k_y}} = J_0(\phi_{k_y} - v_\parallel A_{\parallel,k_y})$ with the Bessel function of the first kind J_0 ($\delta B_\parallel$ is ignored because of considering $\beta_\mathrm{e} \equiv n_{0\mathrm{e}}T_{0\mathrm{e}}/(B_{z0}^2/(2\mu_0)) \ll 1$). $\mathscr{F}$ and $\{a,b\} = (\partial a/\partial x)(\partial b/\partial y) - (\partial a/\partial y)(\partial b/\partial x)$ in the nonlinear term denote the Fourier transformation and the Poisson bracket, respectively. C corresponds to the collision term. (1) is Fourier transformed only in the y direction because it has dependent on the x direction via $k_\parallel = k_y B_y^{\mathrm{eq}}(x)/B_{z0}$. Other notations follow Ref. [4].

Linear phase mixing along the magnetic field line is caused by the second term on the LHS of (1). The nonlinear phase mixing effect due to the FLR effect is expressed by the Bessel function J_0 in the nonlinear term. The first term on the RHS is the source term of the mode destabilization. The saturation amplitude level of the magnetic fluctuations is estimated by the balance between linear and nonlinear terms, but in [2] using the drift-kinetic model, nonlinear phase mixing is not included, and furthermore, $k_\parallel = 0$ is assumed, so the effect of linear phase mixing is also ignored.

In the weakly collisional regime $\nu_\mathrm{ei}/\omega < 1$, the energy is dissipated by the collision term consisting of the velocity derivative since the phase mixing effects create fine-scale structure in velocity space, eventually reaching steady-state turbulence. In this work, we suggest that the saturation amplitude level of magnetic fluctuations in the weakly collisional regime $\nu_\mathrm{ei}/\omega < 1$ increases when the oscillating structure of the electron distribution function, which indicates that phase mixing may be occurring, is formed in velocity space.

Simulation Results

We perform nonlinear gyrokinetic simulations of MT turbulence. The initial system setup of the microtearing simulation in `AstroGK` and the choice of physical parameters follows Ref. [5] and Ref. [3], respectively. In this work, the electron collision frequency $\nu_\mathrm{ee}/\omega_{*\mathrm{e}} = \nu_\mathrm{ei}/\omega_{*\mathrm{e}}$ and temperature gradient $\eta_\mathrm{e} = L_{T_{0\mathrm{e}}}^{-1}/L_{n_{0\mathrm{e}}}^{-1}$ are set as free parameters, where $\omega_{*\mathrm{e}} = (k_y\rho_\mathrm{e}/2)v_{\mathrm{th,e}}(L_{n_{0\mathrm{e}}}^{-1} + L_{T_{0\mathrm{e}}}^{-1})$ with the thermal velocity $v_{\mathrm{th,e}} = \sqrt{2T_{0\mathrm{e}}/m_\mathrm{e}}$.

Figure 1 shows the saturation amplitude of magnetic fluctuations $|\tilde{B}_x|$ normalized by $B_{z0}(\rho_\mathrm{e}/L_{n_{0e}})$ with respect to the electron temperature gradient $\eta_\mathrm{e} = (\rho_\mathrm{e}/L_{T_{0e}})(L_{n_{0e}}/\rho_\mathrm{e})$ for semi-collisional and weakly collisional cases. In the semi-collisional case $\nu_\mathrm{ei}/\omega_{*\mathrm{e}} = 1.56$, the saturation amplitude of magnetic fluctuations follows the scaling of $|\tilde{B}_x|/B_{z0} \sim \rho_\mathrm{e}/L_{T_{0\mathrm{e}}}$. Even in the weakly collisional case $\nu_\mathrm{ei}/\omega_{*\mathrm{e}} = 0.5$, the magnetic fluctuation amplitude is proportional to the temperature gradient. In our linear simulations of MT, the growth rate in $\nu_\mathrm{ei}/\omega_{*\mathrm{e}} = 0.5$ is smaller than that of $\nu_\mathrm{ei}/\omega_{*\mathrm{e}} = 1.56$, but the amplitude of magnetic fluctuations becomes larger in the weakly collisional regime.

To investigate the cause of the larger magnetic fluctuations in the weakly collisional regime,

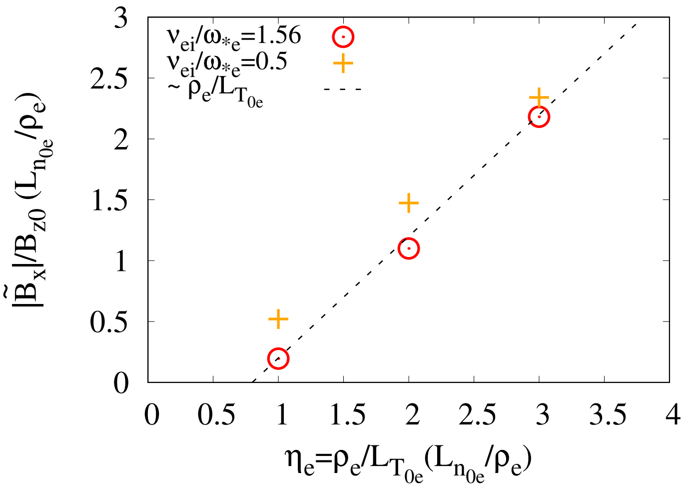


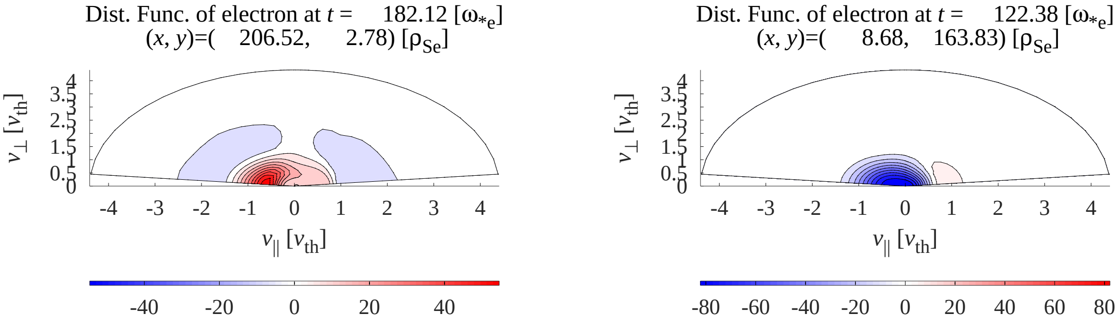
Figure 1: The saturation amplitude of magnetic fluctuations $|\tilde{B}_x|$ normalized by $B_{z0}(\rho_\mathrm{e}/L_{n_{0e}})$ with respect to the electron temperature gradient $\eta_\mathrm{e} = (\rho_\mathrm{e}/L_{T_{0e}})(L_{n_{0e}}/\rho_\mathrm{e})$ for semi-collisional and weakly collisional cases.

we confirm whether phase mixing is occurring in $\nu_\mathrm{ei}/\omega_{*\mathrm{e}} = 0.5$. We plot the velocity space structure of the electron distribution function for $\eta_\mathrm{e} = 2$ in Fig. 2. The velocity structure of $\nu_\mathrm{ei}/\omega_{*\mathrm{e}} = 1.56$ for η_e is also plotted in the right side of Fig. 2 for comparison. The electron distribution function taken where the dissipation rate is large is shown. For $\nu_\mathrm{ei}/\omega_{*\mathrm{e}} = 0.5$, the oscillating structure of the distribution function is formed in both the $v_\|$ and $v_\perp$ directions (left), unlike that of the semi-collisional case (right). The result indicates that the saturation amplitude of magnetic fluctuations may be enhanced due to phase mixing in the weakly collisional regime.

Conclusion

We have performed nonlinear gyrokinetic simulations of MT turbulence in semi-collisional and weakly collisional plasmas in order to investigate the saturation amplitude level of magnetic fluctuations.

We have focused on the effects of linear and nonlinear phase mixing, which is not included in the prediction $|\tilde{B}_x|/B_{z0} \sim \rho_\mathrm{e}/L_{T_{0\mathrm{e}}}$ derived by [2], and have measured $|\tilde{B}_x|/B_{z0}$ against the electron temperature gradient. We have shown that the magnetic fluctuation amplitude in both $\nu_\mathrm{ei}/\omega_{*\mathrm{e}} = 1.56$ and $\nu_\mathrm{ei}/\omega_{*\mathrm{e}} = 0.5$ agrees with the scaling of $|\tilde{B}_x|/B_{z0} \sim \rho_\mathrm{e}/L_{T_{0\mathrm{e}}}$. However, the saturation amplitude of magnetic fluctuations in the weakly collisional regime is larger than that of the semi-collisional case. To investigate the cause of the larger magnetic fluctuations in the weakly collisional regime, we have analyzed the structure of the electron distribution function


Dist. Func. of electron at *t* = 182.12 [ω∗e] (*x*, *y*)=(206.52, 2.78) [ρSe]

Dist. Func. of electron at *t* = 122.38 [ω∗e] (*x*, *y*)=(8.68, 163.83) [ρSe]

Figure 2: Velocity space structure of the electron distribution function in $\nu_{\rm ei}/\omega_{*\rm e} = 0.5, 1.56$ for $\eta_{\rm e} = 2$. The left figure corresponds to $\nu_{\rm ei}/\omega_{*\rm e} = 0.5$, and the right one is for $\nu_{\rm ei}/\omega_{*\rm e} = 1.56$. In the weakly collisional case, the distribution function generates the structure in both the $v_\parallel$ and $v_\perp$ directions, unlike the semi-collisional case.

in velocity space. In the weakly collisional case, the distribution function has the oscillating structure in both the $v_\parallel$ and $v_\perp$ directions, indicating that the magnetic fluctuation amplitude may be enhanced due to the phase mixing effect.

If the saturation amplitude level of magnetic fluctuations is determined due to phase mixing in the weakly collisional regime, the growth rate of MT instability is lower, but the magnetic fluctuation may become higher, and eventually, the electron thermal transport level may be enhanced. To perform quantitative analyses of the effects of phase mixing on the saturation amplitude level of magnetic fluctuation of MT turbulence, the analysis using the Hermite and Laguerre spectrum is required, which is future work.

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