Evolution of nonlinear electrostatic structures in the lunar wake region

Kuldeep Singh[1] and Ioannis Kourakis[1,2,3]

[1] *Department of Mathematics, Khalifa University, Abu Dhabi, UAE*

[2] *Space and Planetary Science Center, Khalifa University, Abu Dhabi, UAE*

[3] *Hellenic Space Center, Leoforos Kifissias 178, Chalandri, GR-15231 Athens, Greece*

Abstract

Inspired by the first flyby of NASA's ARTEMIS mission, which observed the signatures of electrostatics solitary waves in the lunar wake region, we have developed a model for lunar plasma comprising protons, α-particles, an electron beam due to the solar wind (all assumed cold, for simplicity), and suprathermal electrons (following a kappa distribution). The existence of solitary waves has been investigated from the first principles, focusing on the role of the beam and of the spectral index (κ_e). Our findings will help unfold the (mostly unexplored) characteristics of nonlinear waves observed in the lunar wake region.

Introduction. The basic analytical toolbox adopted in the modeling of electrostatic solitary waves (ESW) in plasmas was originally elaborated (independently) by Sagdeev and coworkers (1966) [1] and Washimi & Taniuti (1966) [2] half a century ago. Various theoretical [3] and experimental [4] investigations have since focused on ESWs in different plasma environments, including in space plasmas, e.g. in planetary magnetospheres [5, 6].

This study has been motivated by observations of electrostatic bipolar pulses of frequency in the range $\sim 10-6$ kHz with parallel electric field component (amplitude) around $E_{\parallel} \sim 5-15$ mV/m by the first flyby or the ARTEMIS (Acceleration, Reconnection, Turbulence and Electrodynamics of the Moon's Interaction with the Sun) mission in the lunar wake region. Tao et al. (2012) [7] developed a 1-D Vlasov simulation algorithm to model a four-component lunar wake plasma incorporating protons, alpha (α) particles (i.e. He^{2+} ions), an electron beam and nonthermal (non-Maxwellian) electrons. They observed wave formation in the frequency range $\sim$ (0.1–0.4) f_{pe}, i.e. predominantly an electron beam mode. Rubia et al. (2017) [8] added thermal (pressure) effects, to match those observations with a fluid-theoretical framework.

A multi-fluid plasma model. We have considered a four-component plasma consisting of protons (index p), alpha particles (α), an electron beam (b) with drift velocity $u_{b,0}$ and (inertialess) kappa distributed electrons (e) [9]. Wave propagation is assumed along (i.e. $\parallel$ to) the ambient magnetic field. The former three (inertial) components are described by the equations:

$$\frac{\partial n_j}{\partial t} + \frac{\partial (n_j u_j)}{\partial x} = 0\,, \qquad \frac{\partial u_j}{\partial t} + u_j \frac{\partial u_j}{\partial x} = -\frac{s_j Q_j}{\mu_j}\frac{\partial \phi}{\partial x}\,, \tag{1}$$

where $j = p, \alpha, b$, respectively. The system is closed by Poisson's equation:

$$\frac{\partial^2 \phi}{\partial x^2} = -n_p - Q_\alpha \delta_\alpha n_\alpha + \delta_b n_b + \delta_e \left(1 - \frac{\phi}{\kappa_e - 3/2}\right)^{-\kappa_e + 1/2} . \tag{2}$$

Charge neutrality was assumed at equilibrium, thus: $1 + Q_\alpha \delta_\alpha = \delta_e + \delta_b$. The plasma state variables were normalized by suitable scaling quantities, as follows: $n_j = \tilde{n}_j / n_{p0}$, $u_j = \tilde{u}_j / C_0$ (where $C_0 = (Z_p k_B T_e / m_p)^{1/2}$), $\phi = e\tilde{\phi}/(k_B T_e)$, $x = \tilde{x}/\lambda_{D,e}$ (where $\lambda_{D,e} = \left[\varepsilon_0 k_B T_e / (Z_p e^2 n_{p0})\right]^{1/2}$) and $t = \omega_{p,p} \tilde{t}$ (where $\omega_{p,p} = \left[e^2 Z_p^2 n_{p0} / (\varepsilon_0 m_p)\right]^{1/2}$ is the proton plasma angular frequency). Note that C_0 and $\lambda_{D,e}$ respectively represent the ion sound speed and the electron Debye length in an e-i plasma (and thus *differ* from the analogous quantities in our multifluid plasma configuration). The drifting (streaming) speed of the electron beam was also normalized as $U_{b0} = \tilde{U}_{b0}/C_0$. (The tilde was used above to denote physical quantities, i.e. with dimensions.) We have defined (for $j = p, \alpha, b$) the dimensionless parameters: $Q_j = Z_j / Z_p$ (hence $Q_p = Q_b = 1$, $Q_\alpha = 2$), $\mu_j = m_j / m_p$ (i.e. $\mu_p = 1$, $\mu_\alpha = 4$ and $\mu_b = m_e / m_p \approx 1/1836$) and $s_p = s_\alpha = -s_b = +1$.

Harmonic oscillations of angular frequency ω and wavenumber k obey the dispersion relation

$$\frac{k^2 \left(1 + Q_\alpha^2 \delta_\alpha / \mu_\alpha\right)}{\omega^2} + \frac{k^2 (\delta_b / \mu_b)}{(\omega - k U_{b0})^2} = k^2 + \delta_e \left(\frac{\kappa_e - \frac{1}{2}}{\kappa_e - \frac{3}{2}}\right) . \tag{3}$$

To model stationary profile electrostatic excitations (solitary waves), we transform the above equations into a stationary frame moving at speed v (the solitary wave speed), by adopting the traveling coordinate $\eta = x - Vt$ (i.e. $V = v/C_0$ is the normalized value of the pulse speed). as $\eta \to \pm\infty$, we are led to the energy integral: $\frac{1}{2}\left(\frac{d\phi}{d\eta}\right)^2 + S(\phi, V) = 0$, where the pseudopotential $S(\phi, V)$ reads:

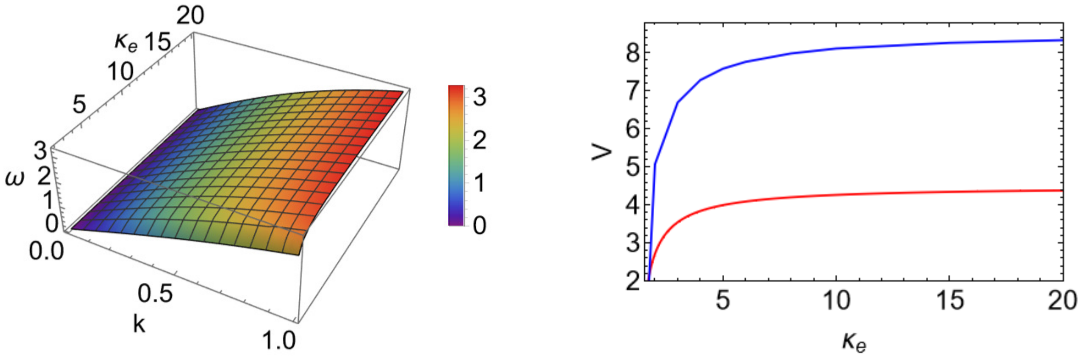
$$S(\phi, V) = V^2 \left[1 - \left(1 - \frac{2\phi}{V^2}\right)^{1/2}\right] + \delta_\alpha \mu_\alpha V^2 \left[1 - \left(1 - \frac{2\phi}{Q_\alpha \mu_\alpha V^2}\right)^{1/2}\right] \tag{4}$$

$$+ \delta_b \mu_b (V - U_{b0})^2 \left[1 - \left(1 + \frac{2\phi}{\mu_b (V - U_{b0})^2}\right)^{1/2}\right] + \delta_e \left[1 - \left(1 - \frac{\phi}{\kappa_e - \frac{3}{2}}\right)^{-\kappa_e + \frac{3}{2}}\right] .$$

For solitary waves to exist, the (so called Sagdeev) pseudopotential function $S(\phi, V)$ must satisfy certain conditions: the origin must be a local maximum, i.e. $S(\phi, V)|_{\phi=0} = S'(\phi, V)|_{\phi=0} = 0$ and $S''(\phi, V)|_{\phi=0} < 0$, and a root ϕ_0 should exist, i.e. $S(\phi, V) = 0|_{\phi=\phi_0}$, such that $S(\phi, V) < 0$ for $0 < |\phi| < |\phi_0|$. (Note that ϕ_0 represents the soliton amplitude.) The first condition imposes:

$$\frac{1 + Q_\alpha^2 \delta_\alpha / \mu_\alpha}{V^2} + \frac{\delta_b / \mu_b}{(V - U_{b0})^2} - \delta_e \left(\frac{\kappa_e - \frac{1}{2}}{\kappa_e - \frac{3}{2}}\right) \leq 0 . \tag{5}$$

Solving for V, this requirement takes the form $V \geq V_s$: the lower bound actually represents the acoustic (sound) speed V_s, as it is obtained from (3); solitary waves are clearly supersonic.



(a) (b)

Figure 1: (a) Plot of the linear (angular) frequency ω of harmonic waves in the $k-\kappa_e$ plane. (b) The soliton existence domain (i.e. the velocity interval where solitary waves may occur) is depicted against the spectral index (κ_e). We have considered $\delta_b = 0.01$, $\delta_\alpha = 0.055$, $\delta_e = 1.1$, $\delta_b = 1.1$ (as imposed from the charge neutrality condition), $u_{b,0} = 0.3$, $\mu_\alpha = 4$, $Q_\alpha = 2$ and $\mu_b = 1/1836$ in these plots.

Careful scrutiny reveals that reality of the state variables $n_j(\phi(V))$ and $u_j(\phi(V))$ (expressions here omitted, for brevity) is violated above 3 successive thresholds (i.e. values of ϕ); given their ordering (viz. $-\frac{\mu_b(V-U_{b0})^2}{2} < \frac{M^2}{2} < \frac{\mu_\alpha M^2}{2Q_\alpha}$, in our case), it suffices to set $\phi_c = -\frac{\mu_b(V-U_{b0})^2}{2}$. Reality is therefore ensured if $S(\phi_c, V_{max}) \geq 0$, implying $V \leq V_{max}$, which can be solved (numerically) for the upper bound V_{max}. The soliton existence region $V \in [V_s, V_{max}]$ depends on the plasma configuration (parameters) as shown in the last Figure and discussed below.

Parametric Analysis. To examine the dynamics of solitary waves in the lunar wake region, we have considered the parameters adopted by Tao et al. (2012) [7]. Fig 1(a) depicts the variation of the angular frequency ω in the $k-\kappa_e$ plane. Note that only one positive root (ω) occurs (for the given parameter set). The frequency and the phase speed of acoustic mode are lower for a nonthermal electron distribution (i.e., for low κ_e) than in the Maxwellian case. For a beam-free plasma (i.e., $\delta_b = 0$), only an IA mode (in the account of the α-particles too) will occur (discussion omitted here). The dependence of the existence domain on the spectral index (κ_e) is shown in Fig 1(b): solitary waves will exist for values between the two curves shown therein. Note that the permitted values for the soliton speed are smaller, for smaller κ_e (value), hence allowing for slower (sometimes misinterpreted as “subsonic") solitary waves.

Fig 2(a) shows the variation of the electrostatic potential profile for different values of the spectral index. Note that only negative polarity solitary structures are formed (the polarity can be determined from $S'''(\phi,V)|_{\phi=0}$). As expected, the amplitude of solitary waves actually increases as κ_e decreases, i.e. the ES potential (disturbance magnitude) is boosted by highly energetic (suprathermal) electrons from the solar wind. Fig 2(b) illustrates the associated potential E-field

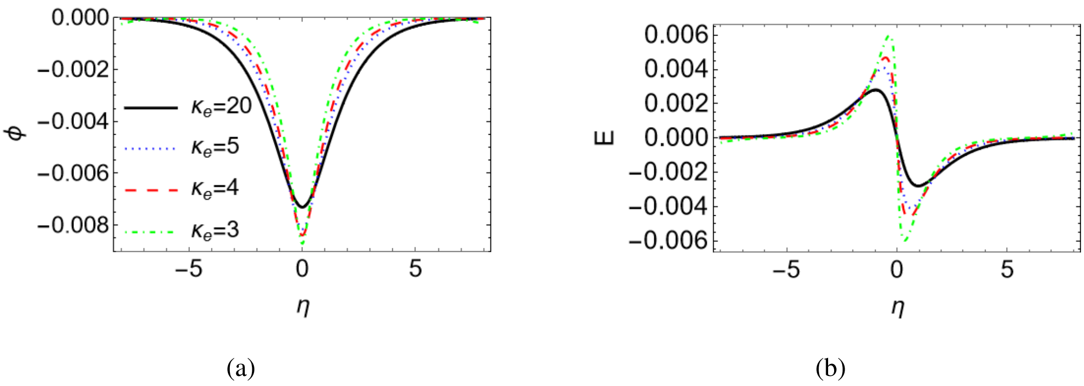


Figure 2: (a) The electrostatic potential pulse (ϕ) and (b) the associated electric field (bipolar pulse) profiles are shown versus the space coordinate η, for different values of κ_e (note the common curve style and color code adopted, provided in the inset label). These plots have been produced by taking $V = 6$, $\delta_b = 0.01$, $\delta_\alpha = 0.055$, $\delta_e = 1.1$, $u_{b,0} = 0.3$, $\mu_\alpha = 4$, $Q_\alpha = 2$ and $\mu_b = 1/1836$.

profile for different values of κ_e.

Conclusions We have investigated the existence and propagation characteristics of electrostatic solitary waves in the lunar wake region, based on a four-component plasma model containing protons, α-particles, streaming electrons (a beam, in account of the solar wind) and suprathermal electrons. Only negative polarity solitary waves are formed. Our study clearly suggests that electrostatic nonlinear waves may occur in the lunar wake region [7]. Our findings will help elucidate the (so far mostly unexplored) characteristics of nonlinear waves in the Lunar wake region. A more detailed study is currently underways and will be reported elsewhere.

Acknowledgments: Authors KS and IK gratefully acknowledge financial support from Khalifa University of Science and Technology, Abu Dhabi UAE via the (internal funding) project FSU-2021-012/8474000352. Author IK also acknowledges financial support from Khalifa University's Space and Planetary Science Center under grant No. KU-SPSC-8474000336.

References

[1] R. Z. Sagdeev, Cooperative phenomena and shock waves in collisionless plasmas, Reviews of Plasma Phys. (Vol. 4), Ed M. A. Leontovich (New York: Consultants Bureau) pp. 23-91 (1966)

[2] H. Washimi and T. Taniuti, Phys. Rev. Lett., **17**, 317 (1966)

[3] F. Verheest and M.A. Hellberg, in Handbook of Solitons: Research, Technology, and Applications (Nova Science Publ.) Eds. S. P. Lang and S. H. Bedore pp. 353-392 (2009).

[4] M. Q. Tran, Physica Scripta, **20**, 317 (1979)

[5] K. Singh, A. Kakad, B. Kakad and I. Kourakis, A & A, **666**, A37 (2022)

[6] B. Kakad, A. Kakad, H. Aravindakshan, I. Kourakis, The Astrophysical Journal **934** (2), 126 (2022)

[7] J. B. Tao et al., J. Geophys. Res., **117**, A03106, (2012)

[8] R. Rubia, S. V. Singh and G. S. Lakhina, J. Geophys. Res., **122**, 9134 (2017)

[9] G. Livadiotis, Kappa Distributions: Theory & Applications in Plasmas (Elsevier, Amsterdam, 2017)