Nonlinear Structures in the Martian Plasma

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Abstract

Mars's induced magnetosphere provides a compelling environment where various plasma processes have been reported, including observations of electrostatic (ES) solitary waves (ESWs). Based on a multi-fluid plasma model, we have explored the dynamics of localized electrostatic structures in the Martian plasma, from first principles.

Introduction. The absence of a natural global magnetic field on Mars results in the solar wind penetrating and interacting directly with the planet's (tenuous) gaseous atmosphere. Due to this interaction, magnetic field tubes of the solar wind bend around the planet and create a magnetized plasma envelope, which eventually behaves as a pseudo-magnetosphere. This induced magnetosphere, is a multi-ion plasma environment where a variety of plasma processes have been reported, including importantly the occurrence of electrostatic (ES) solitary waves (ESWs) [2]. In the martian magnetosheath region, plasma composition is strongly influenced by the solar wind's interaction with heavy ion plasma of planetary and exospheric origin [3, 4].

We have considered a bi-ion / bi-electron plasma comprising O^+ and Cl^- ions and two electron populations (at different temperature), both described by a non-Maxwellian (kappa type) distribution characterized by a long tail in its suprathermal region. Adopting an analytical pseudopotential technique, we have investigated the morphology and propagation characteristics of solitary waves from first principles, based on values observed in Mars's magnetosheath.

Fluid plasma model. We have considered an infinite, homogeneous, collisionless, unmagnetized 4-component plasma consisting of two types of ions of opposite charge and two electron populations. The evolution equations for the ion fluid density n_j and speed u_j (for $j = 1, 2$) are:

$$\begin{aligned} &\frac{\partial n_{i,1}}{\partial t} + \frac{\partial (n_{i,1} u_{i,1})}{\partial x} = 0, && \frac{\partial n_{i,2}}{\partial t} + \frac{\partial (n_{i,2} u_{i,2})}{\partial x} = 0 \\ &\frac{\partial u_{i,1}}{\partial t} + u_{i,1} \frac{\partial u_{i,1}}{\partial x} = -\frac{\partial \phi}{\partial x} && \frac{\partial u_{i,2}}{\partial t} + u_{i,2} \frac{\partial u_{i,2}}{\partial x} = \frac{Q}{\mu} \frac{\partial \phi}{\partial x}, \end{aligned} \tag{1}$$

where $Q = \frac{z_{i,2}}{z_{i,1}}$, and $\mu = \frac{m_{i,2}}{m_{i,1}}$ denote the ion-to-ion (absolute) charge and mass ratio(s) and ϕ is the (normalized) ES potential. The system is closed by the (dimensionless) Poisson equation:

$$\frac{\partial^2 \phi}{\partial x^2} = -n_{i,1} + Q\delta n_{i,2} + (1 - Q\delta)(n_{e,c} + n_{e,h}), \tag{2}$$

where $\delta = \frac{n_{i,2,0}}{n_{i,1,0}}$. Both electron components are assumed to be kappa-distributed, therefore

$$n_{e,c} = \zeta \left[1 - \frac{\beta\phi}{(\kappa_{e,c} - \frac{3}{2})T_{e,c}}\right]^{-\kappa_{e,c}+\frac{1}{2}}, \qquad n_{e,h} = \nu \left[1 - \frac{\phi}{(\kappa_{e,h} - \frac{3}{2})T_{e,h}}\right]^{-\kappa_{e,h}+\frac{1}{2}}. \tag{3}$$

where $\beta = \frac{Teh}{T_{ec}}$, $\zeta = \frac{n_{e,c,0}}{n_{e,0}}$ and $\nu = \frac{n_{e,h,0}}{n_{e,0}} = 1-\zeta$. In the above equations, the ion number densities are scaled by the respective equilibrium densities ($n_{i,j,0}$, respectively, for $j = 1,2$), while the electron densities are scaled by the total (equilibrium) electron density $n_{e,0} = n_{e,c,0} + n_{e,h,0}$. The ion fluid speed variables were scaled by the characteristic speed (scale) $c_0 = \left(\frac{z_{i,1}K_B T_{e,h}}{m_{i,1}}\right)^{1/2}$, while time and space were scaled by the (primary ion) plasma period $\omega_{p,i,1}^{-1} = \left[\frac{n_{i,1,0}(z_{i,1}e)^2}{\varepsilon_0 m_{i,1}}\right]^{-1/2}$ and by the characteristic length $\lambda_0 = c_0/\omega_{p,i,1} = \left[\frac{\varepsilon_0 K_B T_{e,h}}{n_{i,1,0} z_{i,1} e^2}\right]^{1/2}$, respectively. Finally, the ES potential was scaled by $\frac{K_B T_{e,h}}{e}$. The ion fluids were assumed "cold", for analytical simplicity.

Solving the above fluid equations in the stationary wave frame, $\eta = x - Vt$, where V is the normalized solitary wave (pulse) speed (i.e. $V = v_{SW}/c_0$) and adopting vanishing boundary conditions for the density and fluid speed variables, one obtains $\frac{1}{2}\left(\frac{\partial\phi}{\partial\eta}\right)^2 + S(\phi) = 0$, where $S(\phi)$ is the – so called Sagdeev [5, 6] – pseudopotential function

$$\begin{aligned} S(\phi) = (1-Q\delta)\Bigg\{\frac{\zeta}{\beta}\left[1-\left(1-\frac{\beta\phi}{(\kappa_{e,c}-\frac{3}{2})}\right)^{-\kappa_{e,c}+\frac{3}{2}}\right] + \nu\left[1-\left(1-\frac{\phi}{(\kappa_{e,h}-\frac{3}{2})}\right)^{-\kappa_{e,h}+\frac{3}{2}}\right]\Bigg\} \\ + V^2\left[1-\left(1-\frac{2\phi}{V^2}\right)^{1/2}\right] + \delta V^2\mu\left[1-\left(1+\frac{2Q\phi}{\mu V^2}\right)^{1/2}\right]. \end{aligned} \tag{4}$$

Existence conditions for solitary waves. The procedure to investigate the existence conditions for localized solutions follows closely the standard pseudopotential method [6], as applied to a bi-ion problem e.g. in Ref. [1]. The superacoustic condition – cf. Eq. (17) in [1] – reads:

$$\frac{\partial^2 S}{\partial\phi^2}\Big|_{\phi=0} = \frac{1}{V^2} + \frac{Q^2\delta}{\mu V^2} - (1-Q\delta)\,(\zeta\beta C_{ec} + \nu C_{eh}) < 0 \tag{5}$$

where $C_{ec} = \frac{\kappa_{ec}-\frac{1}{2}}{\kappa_{ec}-\frac{3}{2}}$ and $C_{eh} = \frac{\kappa_{eh}-\frac{1}{2}}{\kappa_{eh}-\frac{3}{2}}$. One thus finds a lower bound (say, V_1) for the velocity of the localized structure, equal to

$$V > V_1 = \left(1 + \frac{Q^2\delta}{\mu}\right)^{\frac{1}{2}} \left[(1-Q\delta)(\zeta\beta C_{ec} + \nu C_{eh})\right]^{-\frac{1}{2}}. \tag{6}$$

This is actually the sound speed, in this particular plasma configuration (as one might easily show by linear analysis): solitary waves are therefore supersonic (superacoustic).

The (obvious) requirement of reality of all of the state variables imposes: an upper limit (say, V_2) for the pulse speed. This is found (numerically) by imposing: $S\left(\frac{V^2}{2}\right) \geq 0$ (for $\phi > 0$) and $S\left(-\frac{\mu V^2}{2Q}\right) \geq 0$ (for $\phi < 0$): both of these condition lead to an inequality in the form $V \leq V_2^{(\pm)}$.

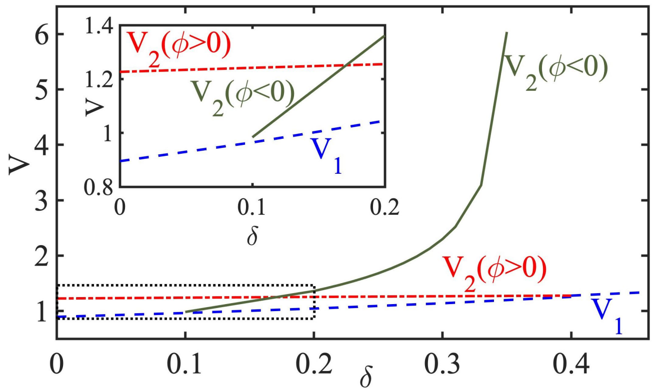

Figure 1: Existence diagram for solitary waves: ESWs occur for $V \in [V_1, V_2]$. Note that two upper curves (V_2) exist, for positive or/and negative polarity ESWs. The inner panel shows a zoom-in view of the interval $\delta \in [0-0.2]$. Here, $\zeta = 0.01$; $Q = 2.216$, $z_{i1,2} = 1$, $\nu = 0.99$, $\kappa_{e,c} = 4$, $\kappa_{e,h} = 6$ and $\beta = 2.7$.

Solitary wave existence requires $V \in [V_1, V_2^{(\pm)}]$ (where the upper/lower sign is understood, for positive/negative polarity ϕ pulses to exist.) One may thus identify regions where a) positive (only), b) negative (only), or c) either negative or positive (coexistence) pulses may occur. This is visible in Fig. 1, showing the dependence of the curves V_1 and $V_2^{(\pm)}$ on the bi-ion relative concentration, via δ. Note that *only* negative polarity is the only possibility for large δ, i.e. when the secondary (negative) ion component is dominant.

Nonlinear Analysis. To show the existence of different types of nonlinear structures, we have adopted a representative parameter set, combined with three convenient values for the pulse speed V. In Fig. 2, we have plotted the pseudopotential S (left panel) in addition to the E-field bipolar waveforms obtained for this particular parameter set (of values). Note that both negative and positive polarity pulses exist in this case. In the former case (inverse polarity), only “standard" bipolar forms occur, whereas in the latter case (positive polarity), considering higher V, regular E-field forms are replaced by flat-top solitary waves (with a characteristic elongated “offset bipolar" E-field structure), subsequently giving rise to large-amplitude super-solitary waves (with their characteristic curly profile). The existence condition for these structures are explained in [1]. Such stretched bipolar pulses are often called as offset bipolar pulses. Figure. 2(i) curve (III) represents the wiggled bipolar pulse.

The observational data presented in [2] included an electron temperature around $3-4600$ eV, along with electron density ≈ 15 cm^{-3}. The oxygen (O^+) density was measured around ≈ 15 cm^{-3}. For the remaining parameters, we have adopted a typical parameter set, assuming $T_{e,h} = 30$ eV, $n_{i,10} = 15$ cm^{-3} and $n_{i,2,0} = 5.1$ cm^{-3}, in addition to $\beta = 2.7$ and $\zeta = 0.01$ (ad hoc values). Considering representative values, say, $\kappa_{e,c} = 4$ and $\kappa_{e,h} = 6$, for the low/high