Resonant axisymmetric modes in tokamak plasmas

F. Porcelli[1,2], T. Barberis[1], D. Banerjee[1], Charlson C. Kim[3], and A. Yolbarsop[2]

[1] *Department of Applied Science and Technology, Polytechnic University of Turin, Italy*

[2] *School of Nuclear Science and Technology, USTC, Hefei, 230022, China*

[3] *SLS2 Consulting, San Diego, California 92107, USA*

Axisymmetric modes (toroidal mode number $n = 0$) are natural modes of oscillation in elongated tokamak plasmas. These modes are singular at magnetic X-points - a circumstance that is likely to drive axisymmetric current sheets in the vicinity of an X-point and along the divertor magnetic separatrix. Given a generic perturbation, ξ, the resonant condition is expressed by $\mathbf{B}_{\rm eq}\cdot\nabla\xi = i(nB_T/R)\xi + \mathbf{B}_{\rm p}\cdot\nabla\xi = 0$, which is satisfied for $n = 0$, since the equilibrium poloidal field $\mathbf{B}_{\rm p}$ vanishes at the X-point. Current sheets forming near X-points are likely to have an impact on ELM stability and on transport properties at the plasma edge. Furthermore, $n = 0$ oscillations are global modes, with a frequency on the Alfvén range, and as such, they can resonate with Mev ions in a tokamak, giving rise to a new type of fast ion instability. For these reasons, we believe that the study of axisymmetric modes in tokamak plasmas deserves more attention than it has received so far.

Following the pioneering works by Laval, Wesson, and coworkers [1, 2], the analytic theory of $n = 0$ vertical modes has been reassessed in a series of recent publications [3, 4, 5, 6, 7]. Considering a *straight tokamak* equilibrium with an elliptical cross-section and the well-known reduced ideal-MHD model, one can derive a cubic dispersion relation for a *limiter-like* configuration, i.e., assuming that the plasma density drops to zero before the magnetic separatrix, so that magnetic X-points are in the vacuum. The dispersion relation reads:

$$\gamma^3 + \gamma^2 \frac{1}{\tau_\eta}\frac{1}{1-\hat{e}_0 D} + \gamma\omega_0^2 + \omega_0^2 \frac{1}{\tau_\eta}\frac{1}{1-D} = 0. \tag{1}$$

When the wall-parameter D is larger than unity, the three roots of Eq.1 are:

$$\omega \approx \pm\omega_0 - i\frac{1}{2\tau_\eta}\frac{D}{(D-1)} = \pm\omega_0 - i\gamma_\eta \tag{2}$$

$$\gamma = \frac{1}{(D-1)\tau_\eta} \tag{3}$$

where the resistive wall time scale τ_η is proportional to the wall resistivity, $\omega_0 \approx e_0^{1/2}\tau_A^{-1}\sqrt{D-1}$ is the mode frequency, e_0 is the ellipticity of the plasma boundary, and τ_A the poloidal Alfvén time. The parameter D depends on plasma elongation and on the distance of the wall from the plasma [6]. If the wall is moved away from the plasma, D drops below unity and the vertical displacement becomes ideal-MHD unstable. Thus, an ideal wall can provide passive feedback stabilization when sufficiently close to the plasma. The residual instability, growing on the resistive wall time scale, can be suppressed by active feedback stabilization. However, it is shown in Ref. [6] that the resistive growth rate can be significantly faster, scaling with fractional powers of wall resistivity, when $D \approx 1$, i.e., close to ideal-MHD marginal stability, thus posing more stringent conditions for active feedback stabilization.

The analytic theory for the *limiter-like* configuration has been tested numerically using the NIMROD code [8]. Fig. 1 shows a comparison between analytic and numerical values of ω^2 for vertical and *horizontal* $n = 0$ displacements, where *horizontal* indicates the direction along the minor axis of the elliptical cross-section. A perfectly conducting wall is assumed. Clearly, horizontal displacements are always stable ($\omega^2 > 0$), while vertical displacements can become ideal-MHD unstable if the parameter b/b_w is smaller than a critical threshold, with b the major semi-axis of the elliptical plasma boundary and b_w that of the confocal elliptical wall. The comparison between analytic theory and NIMROD results is highly satisfactory (see [7, 9]).

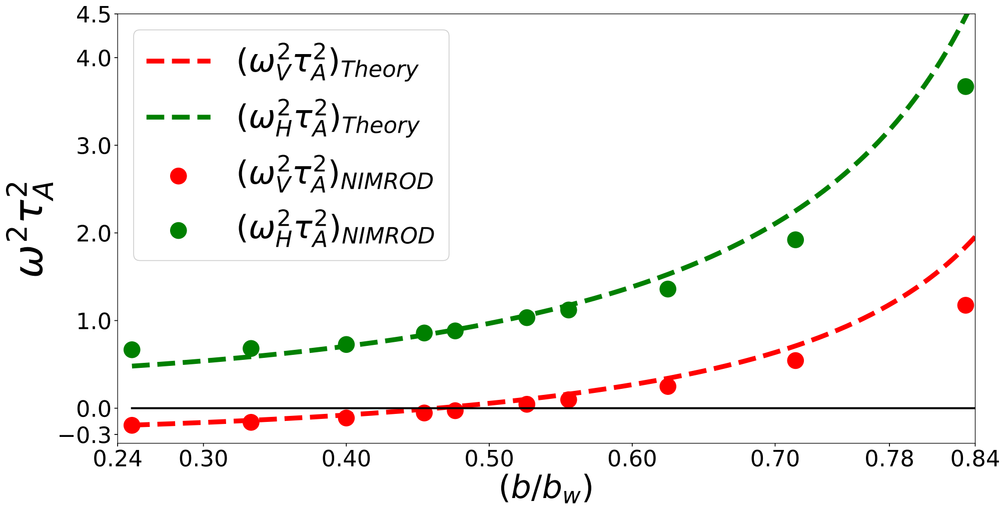

Figure 1: Plots of mode frequencies (ω^2), normalized to the relevant Alfvén time, as functions of b/b_w; dashed -lines: analytic solutions, dots: NIMROD numerical results. V stands for vertical, and H for horizontal modes; b and b_w are the major semi-axes of the elliptical plasma boundary and of the confocal elliptical wall, respectively.

Considering the more realistic *divertor-like* configuration, where the hot plasma extends all the way to the magnetic separatrix, analytic work reported in Refs. [3, 4] showed that X-point currents can completely suppress the ideal-MHD vertical instability in the no-wall limit. The stabilization mechanism is a direct consequence of the resonant nature of magnetic-X-points, which in the ideal-MHD limit can be regularized by imposing the flux-freezing constraint on these points. As a consequence, current sheets at the X-points, extending along the separatrix, develop. In the ideal-MHD limit, these current sheets have vanishing width and are capable of exerting a force pushing back the plasma in its vertical motion. As far as we are aware, Ref. [3] is the first article where the mathematics of the X-point singularity with respect to $n = 0$ modes in the ideal-MHD limit was established. Numerical work is in progress to extend NIMROD

Figure 2: Contour visualization of the perturbed current density, demonstrating the development of perturbed current sheets in close proximity to magnetic X-points and along the magnetic separatrix and also at the elliptical boundary due to the vertical motion of plasma, a feature which corroborates the analytic results.

simulations to the *divertor-like* configuration. However, even in the *limiter-like* configuration, X-point currents are observed in simulations. In fact, the NIMROD code adopts a low-density, high-resistivity halo plasma instead of a vacuum, as assumed in analytic theory. This is a welcome circumstance, as the occurrence of axisymmetric currents localized near the X-points, carried by the halo plasma, are clearly visible in simulation results, see Fig. 2. This is preliminary evidence that X-point currents are stabilizing, although so far, only a relatively small reduction of the linear growth rate of vertical modes due to these halo-plasma localized currents