Development of dynamically coupled simulation for global turbulent transport and profile formation

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Scope of this study

In magnetically confined fusion plasmas, the quantitative prediction of turbulent transport and pressure profile is one of the critical issues. Many numerical studies in terms of the first–principle–based global nonlinear gyrokinetic simulations (such as[1, 2]) had been performed, including the effect of electromagnetic and multiple particle species. Due to the requirement of huge computational costs, the global gyrokinetic simulations are often limited to analyze various conditions of the plasma and the confinement magnetic field. In order to explore optimal or novel confinement states and the relevant operation scenarios for fusion burning plasmas , the development of a more efficient global simulation framework with a reduced turbulent transport model is indispensable.

To this end, the dynamically coupled simulation(DCS) is developed based on the direct multiscale coupling between a 1D global transport solver and radially distributed 5D local gyrokinetic simulations. The numerical framework is similar to the earlier work[3], but a novel reduced transport model[4] is combined in this study. The newly constructed reduced transport model can accurately reproduce the nonlinear turbulent transport flux and the zonal–flow intensity only by linear gyrokinetic simulation results.

Construction of reduced turbulent transport model

The reduced turbulent transport model, which is used in the DCS, is presented in this section. The modeling is performed in 2 steps, following the earlier works[5, 6]. First, the nonlinear functional relation(NFR) among the turbulent transport, the turbulence intensity, and the zonal–flow intensity from the nonlinear gyrokinetic simulation is identified[7]. Second, the further modeling for the turbulent intensity and the zonal–flow intensity is performed by using the linear gyrokinetic simulation results, and then these models are combined with the NFR.

The turbulent transport(heat diffusivity) in the gyro–Bohm unit $\chi_{\mathrm{i}}/\chi_{\mathrm{i}}^{GB}$, turbulence intensity $\mathcal{T}$, and zonal–flow intensity $\mathcal{Z}$ are considered in NFR. These quantities are obtained from nonlinear gyrokinetic simulation for tokamak ITG turbulence with adiabatic electrons[8], where the turbulence intensity $\mathcal{T}$ and the zonal–flow intensity $\mathcal{Z}$ are defined as follows:

$$\mathcal{T} = \frac{1}{2} \sum_{k_x, k_y \neq 0} \langle |\delta\phi_{k_x,k_y}|^2 \rangle, \quad (1)$$

$$\mathcal{Z} = \frac{1}{2} \sum_{k_x} \langle |\delta\phi_{k_x,k_y=0}|^2 \rangle. \quad (2)$$

Here, $\boldsymbol{k}_\perp = (k_x, k_y)$ and $\delta\phi_{k_x,k_y}$ are the perpendicular wavenumber vector and the electrostatic potential fluctuation in Fourier space. These quantities are averaged in time over the statistical steady states.

The functional form of NFR is defined as:

$$\frac{\chi_\mathrm{i}}{\chi_\mathrm{i}^\mathrm{GB}} \sim F^\mathrm{NFR}(\mathcal{T}, \mathcal{Z}) = \frac{C_1 \mathcal{T}^\alpha}{1 + C_2(\mathcal{Z}^\beta / \mathcal{T}^\xi)}. \quad (3)$$

The heat diffusivity is proportional to the turbulence intensity $\mathcal{T}$, like the quaslinear transport model. Furthermore, the transport suppression effect caused by the zonal flows is represented by the form of $(\mathcal{Z}^\beta / \mathcal{T}^\xi)$ in the denominator. This functional form of NFR is not unique but still satisfies the fundamental phenomenological requirements. Then, $(C_1, C_2, \alpha, \beta, \xi)$ in the NFR are the regression parameters to be determined by using the mathematical optimization techniques in Ref. [7].

Next, the further modeling of $\mathcal{T}$ and $\mathcal{Z}$ provides a reduced transport model. Following earlier works[5, 6], $\mathcal{T}$ and $\mathcal{Z}$ are approximated by the quantities obtained only by the linear gyrokinetic simulations, such as the growth rate γ_{k_x,k_y} of the ITG instability and the zonal-flow response function $\mathcal{R}_{k_x}(t) := \langle \delta\phi_{k_x,k_y=0}(t) \rangle / \langle \delta\phi_{k_x,k_y=0}(0) \rangle$. In order to approximate $\mathcal{T}$ and $\mathcal{Z}$ by these quantities, the mixing–length diffusivity $\mathcal{L}$ and the zonal–flow decay time τ_ZF are introduced:

$$\mathcal{L} := \sum_{k_y} \frac{\gamma_{k_x=0,k_y}}{k_y^2}, \quad (4)$$

$$\tau_\mathrm{ZF} := \int_0^{\tau_\mathrm{f}(\gamma_\mathrm{max})} dt \mathcal{R}_{k_x}(t), \quad (5)$$

where the upper limit of the integral interval $\tau_\mathrm{f}(\gamma_\mathrm{max})$, which is a function of the maximum growth rate γ_max, is given by a typical time scale of the turbulence correlation, i.e., $\tau_\mathrm{f} = C/\gamma_\mathrm{max}$ with a constant C. In the earlier works[5, 6], τ_ZF is independent of the temperature gradient by assuming a constant τ_f Since $\mathcal{Z}$ has a temperature gradient dependence, the quantity τ_{ZF} should also have the temperature gradient dependence. Indeed, the temperature gradient dependence was introduced in τ_{ZF} through the integral upper limit $\tau_{\mathrm{f}}(\gamma_{\max})$.

The model functional form of the $\mathcal{T}$ and $\mathcal{Z}$ are determined to reproduce the trend of the data, which is shown in Ref. [4]. Then, substituting these model functions into Eq. (3), a reduced transport model for the ITG–driven turbulent heat diffusivity is expressed as follows:

$$\frac{\chi_{\mathrm{i}}}{\chi_{\mathrm{i}}^{\mathrm{GB}}} \sim \frac{\chi_{\mathrm{i}}^{\mathrm{model}}}{\chi_{\mathrm{i}}^{\mathrm{GB}}}(\mathcal{L}, \tau_{\mathrm{ZF}}) = \frac{\Theta_1 \mathcal{L}^{\Theta_2} \exp\left(\Theta_3 {\tau_{\mathrm{ZF}}}^{\Theta_4}\right)}{1 + \Theta_5 \mathcal{L}^{\Theta_6} \exp\left(\Theta_7 {\tau_{\mathrm{ZF}}}^{\Theta_4}\right)\left[\mathcal{H}(\tau_{\mathrm{ZF}})\right]^{\Theta_8}}. \tag{6}$$

Here, $\Theta_{\mathrm{i}}(\mathrm{i} = 1, 2, ..., 8)$ are parameters to be calculated by the combinations of the regression parameters[4]. Figure 1 shows the comparison between the constructed transport model and the previous model[5]. The newly constructed transport model indicates smaller regression error as $\sigma_{\mathrm{model}} = 0.157$, compared to that in previous model with $\sigma_{\mathrm{model}} = 0.618$, where σ_{model} is evaluated by root–mean–square error. It should also be emphasized that the present transport model enables us to accurately reproduce not only the turbulent heat diffusivity, but also the zonal flow intensity.

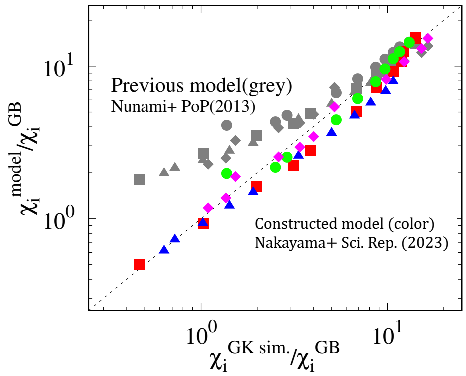

Figure 1: *Comparison of prediction accuracy between the constructed model(color plots) and previous models(grey plots)[5].*

Dynamically coupled global transport simulation

For the analysis of fusion burning plasma, the Dynamically Coupled Simulation(DCS) is developed. As shown in Fig. 2, 1D global transport solver and radially distributed 5D local gyrokinetic simulations are combined by Multiple Program Multiple Data(MPMD) parallelization. It can perform large-scale nonlinear analysis in direct coupling with nonlinear gyrokinetic–simulation. Also, combined with the reduced transport model in the previous section, it allows for faster simulation. It is noted that MPMD

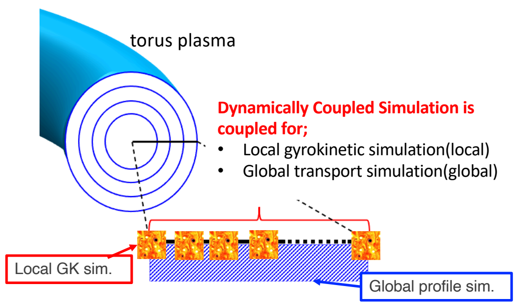

Figure 2: *Schematic of dynamically coupled simulation, composed of global transport solver and radially distributed local gyrokinetic simulations.*