Progress Report on the Multi-temperature Generalized Zhdanov Closure

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Introduction and Challenges

Impurity transport in the SOL/edge of current and next-gen fusion devices represents a formidable challenge in magnetic confinement fusion research. The plasma in the SOL/edge is generally modelled through fluid codes for the main plasma coupled to a kinetic code for neutral species. Examples of such codes include Soledge3x-EIRENE and SOLPS-ITER both coupled to EIRENE, the kinetic code package for neutrals. On the fluid modelling side, impurities cannot generally be treated as a trace species and therefore, one cannot resort to calculating friction and thermal forces on them as if they were, where the forces dictate the transport. Impurities are also at multiple states of ionization, with each ionization state behaving as its own component species. Such plasmas in general are referred to as multi-component plasmas. For a simple ion-electron plasma, with the ion being of a fixed charge state, there are prescriptions for friction and thermal already available, such as by Braginskii[1]. However, it assumes that the species temperatures are the same, which may not necessarily be true near the divertor region of current fusion devices. One more requirement to be mentioned is that, for current plasma fluid code packages, which simulate the continuity, momentum and temperature equations fully, can be missing accurate expressions pertaining to the divergence of viscous-stress in the momentum equation, and the divergence of the heat-flux in the energy equation, in addition to requiring precise collisional terms from the RHS. Therefore, a “closure” is needed in order to supply expressions for these quantities.

Assuming the linear transport regime for such missing quantities[2], a closure scheme was prescribed by Zhdanov et al[3], which would be capable of prescribing the collisional forces, the heat-fluxes, and the viscous-stress tensors for multi-component plasmas. The closure scheme assumes a 21N-moment approach, 1 moment each for the density and temperature, 3 moments for the flow velocity, heat-flux, and a higher-order heat-flux, and 5 moments for the viscous-stress tensor and its higher-order variant. Generally, these many moments represent a significant improvement in the precision of collisional forces and transport coefficients. However, the Zhdanov closure as originally prescribed, tends to mix the single-temperature and multi-temperature approaches, and as a result there is a hidden assumption that the temperatures of all species are near a common temperature value. There is also another assumption that the flows of all species are near each other. The Zhdanov closure has been implemented in a variety of numerical SOL/edge fluid codes, such as Soledge3x-EIRENE[4] and SOLPS-ITER[5, 6]. The differences in temperatures do not affect codes like SOLPS-ITER which solve one equation for total plasma energy, but may play a role in Soledge3x-EIRENE where different species can be at different temperatures.

Progress

To address the subtlety of multi-temperature plasmas, with relevance to Soledge3x-EIRENE simulations, we first derived the collisional coefficients, i.e. the coefficients which multiply the moments on the RHSs of the balance equations, starting from the linearized Boltzmann operator with the species flows being close to common flows, relaxing the assumption on the temperatures of the species[7]. The collisional coefficients are essentially found to be linear combinations of the product of a coefficient term $A^{pqrl}_{\alpha\beta/\alpha\alpha}$, which depends purely on the masses and temperatures of the species, with $\Omega^{lr}_{\alpha\beta}$, the Chapman-Cowling integrals[8], which depend on the potential of interaction, for which we chose the Coulomb potential (where α, β represent the species indices). They were found to agree with the lower-order multi-temperature coefficients provided by Zhdanov et al, at agreed with the higher-order single-temperature coefficients when the temperatures were equal. Furthermore, we found that in the vicinity of unity temperature ratio, the collision coefficients agreed very well with the single-temperature ones, but for temperature ratios of even between 0.5-2, we found significant divergence (50-200%), a trend also found in a limited 13N-moment calculation of the viscosity and thermal conductivities.

In the next part of the work, we re-derived and verified the expressions from Zhdanov closure, which is a Sonine-Hermite polynomial based moment method. We delineated the assumptions and investigated the Knudsen number (K_n) based approximation method, arriving upon a closure scheme generalized to any number of moments[9]. In short, the plasmadynamical moments $(\rho_\alpha, \rho_\alpha \mathbf{w}_\alpha, n_\alpha k T_\alpha)$ and their space and time derivatives are considered of a zeroth order in Knudsen number K_n, and just the higher-order moments are considered to be of first order in K_n. The first and higher-order time derivatives and the gradients of higher-order moments, are all considered to be of or smaller than the order $(K_n)^2$. This leads to a linear system of balance equations for the higher-order moments rendered solvable by linear algebra. One also obtains the friction and thermal forces in their usual forms, in terms of species flows and temperature gradients respectively.

We also studied the convergence of the multi-temperature scheme with addition of more moments. It was found that that there are diminishing contributions to the parallel transport coefficients and the collisional forces beyond the 21N-moment scheme (convergence to two decimal places with 21N-moments). However, the electron parallel thermal conductivities may be susceptible to a slow convergence and may require more moments when mid-weight impurities such as argon are present at significant densities of the main ion density. It was also found that the difference between the single-temperature and multi-temperature transport coefficients can be significant across a scan of temperature ratios from 0.5-2 (between 10-80%, with the higher differences for mid-weight impurities). A similar trend was observed for the convergence of parallel collisional forces as well, with mid-weight impurity thermal forces converging relatively slowly. This keeps open the question of needing more moments for modelling mid-weight impurities at significant density fractions. Generally, for any species at significant density fraction, it was found prudent to recommend the multi-temperature scheme when the temperatures of different species remain different.

A generalization of the scheme to the Burnett approximation was provided as well, and an extension of the scheme in the drift-approximation as well. The first can be generalized for any number of moments, but the second is limited to 21N-moments, as the neoclassical approximations tend to be at the 13N-moment level for the perpendicular averaged flows. More work is necessary to make the Knudsen number and drift small parameter ordering schemes consistent with each other. In addition to this, the 21N-moment multi-temperature collision coefficient expressions were published for the general community[10]. The implementation of the multi-temperature closure for the collisional forces is in progress for Soledge3x-EIRENE.

Outlook

The chief topic would be to relax the flow assumption on the collisional coefficients. This is because one usually finds the flows near the boundaries diverging as a result of the Bohm criterion. Therefore the closure has to be immune to it. However, such a flow generalization may also entail non-linear terms of first-order in Knudsen number, and collision coefficients themselves taking a tensorial form. It would be difficult to calculate such terms, firstly, and secondly, to eliminate such terms under the Knudsen number approximation scheme. They also break the convenient linearity of the Zhdanov closure. Given the significant algebraic barriers, we have to weigh the effort needed for such a generalization against against any physics benefit we may obtain from it. Furthermore, the validity of Bohm criterion for multi-component plasmas is also an open question. The algebraic efforts will also have to be weighed against results from contemporary experimental research on multi-component sheaths[11] to see whether the relaxation of the flow assumption is really required for simulating SOL/edge plasmas, especially near the divertor targets.

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