First-order shift formula of stable and unstable manifolds under perturbation and its application in magnetic confinement fusion

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Introduction

In the established theory [1, 2] of the global structure of three-dimensional (3D) magnetic fields, we had derived *the first-order shift formula of X/O-cycles under perturbation* ($\delta\mathscr{B}$) [1], based on which *the first-order shift formula of stable and unstable manifolds under perturbation* is further deduced in this work. These two formulae provide a new perspective for controlling the shape of magnetically confined plasma by applying them to the vacuum magnetic fields induced by various magnetic coils. Moreover, it is feasible to control the width of chaotic layers at the plasma edge and island chains using these formulae. Notably, among all the "perturbing" fields, the time derivative of a realistic field, $\partial\mathscr{B}/\partial t$, can be considered a peculiar one in the formulae (*i.e.*, substituted for the perturbing field $\delta\mathscr{B}$), which yields the shift velocities of X/O cycles, stable and unstable manifolds. It should be noted that neither the perturbation field $\delta\mathscr{B}$ nor the field to be perturbed needs to be *axisymmetric* or *divergence-free* in this theory, which has the potential to be generalized to general N-dimensional dynamical systems.

Deduction

To obtain the first-order shift formula of stable and unstable manifolds under perturbation, one simply needs to consider the arc length change during field line tracing based on *orbit shift under perturbation formula* [1], which is repeated below:

$$\frac{\partial}{\partial\phi}\delta\boldsymbol{X}_{\mathrm{pol}}[\mathcal{B};\Delta\mathcal{B}](\phi) = \overbrace{\frac{\partial(R\boldsymbol{B}_{\mathrm{pol}}/B_\phi)}{\partial(R,Z)}(\phi)}^{:=\mathbf{A}(\phi)}\ \delta\boldsymbol{X}_{\mathrm{pol}}[\mathcal{B};\Delta\mathcal{B}](\phi) + \overbrace{\begin{bmatrix} R/B_\phi & 0 & -RB_R/B_\phi^2 \\ 0 & R/B_\phi & -RB_Z/B_\phi^2 \end{bmatrix}(\phi)\underbrace{\left(\frac{\delta\left[B_R,B_Z,B_\phi\right]^T}{\delta\mathcal{B}}\Delta\mathcal{B}\right)}_{\text{simply }=[\Delta B_R,\Delta B_Z,\Delta B_\phi]^T}}^{:=\boldsymbol{q}(\phi)}$$

$$\frac{\partial}{\partial\phi} \Downarrow \frac{\mathrm{d}}{\mathrm{d}\phi}, \quad (\phi) \Downarrow (s(\phi),\phi), \quad (\phi) \Downarrow (s,\phi)$$

$$\frac{\mathrm{d}}{\mathrm{d}\phi}\delta\boldsymbol{X}_{\mathrm{pol}}[\mathcal{B};\Delta\mathcal{B}](s(\phi),\phi) = \left(\frac{\mathrm{d}s}{\mathrm{d}\phi}\frac{\partial}{\partial s} + \frac{\partial}{\partial\phi}\right)\delta\boldsymbol{X}_{\mathrm{pol}}[\mathcal{B};\Delta\mathcal{B}](s,\phi)$$

The blue colored symbols in the formula are what need to be changed if the arc length s is considered (so that the formula applies to a (un)stable manifold instead of a general orbit).