A quasi-2D approach for simulating corona discharges in air

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Introduction

Corona discharges, especially for ionic propulsion, have been studied extensively. One of their applications are EHD thrusters, that generate thrust through momentum-exchange collisions between partially ionized plasma and neutral gas, creating ionic wind. Models of these devices often involve two regions: ionization (where ions form due to an electric field) and drift (where ions accelerate the gas). While comprehensive models should account for the interaction between discharge and gas flow, most current models only consider a one-way coupling. This paper examines the basic mechanisms of the EHD force, focusing on the discharge region. Traditional 1D simulations suit wire-cylinder setups but not wire-to-wire geometries. The quasi-2D approximation used in [1] offers a computationally efficient alternative for wire-to-wire geometries by restricting the domain to harmonic electric field lines. We used a drift-diffusion-reaction model to simulate corona discharge in wire-wire geometry employing the quasi-2D approximation. Our findings show that the EHD force density is greatly affected by the kinetic processes in the model, leading to different thrust results in ionic thruster applications.

Physical and Numerical Model

The discharge model relies on the time-dependent drift-diffusion approximation. Each plasma species' evolution follows the continuity equation:

$$\frac{\partial n}{\partial t} + \nabla \cdot \Gamma = S. \tag{1}$$

Here, n is the number density, S is the source term for particle production, and Γ is the flux density given by $\Gamma = -D\nabla n + \text{sign}(q)\,\mu E n$, where D and μ are the diffusion coefficient and electrical mobility, respectively, q is the species charge, and E is the electric field. For neutral species, mobility is zero. The diffusion coefficient follows the Einstein relation: $D = \mu k_B T/e$.

In our model, air is treated as an ideal gas at atmospheric pressure, and the local field approximation (LFA) is used, expressing electron temperature as a function of the local reduced electric field E/N, where N is the gas number density. The ion and neutral gas temperatures are assumed constant at 300 K. The source term S is computed using two models: a 3-species, 4-reactions Townsend model [2] and a 6-species, 13-reactions model used in [3]. The latter, from here on out referred to as the Parent model was also employed in a DBD discharge [4].

The electric field E is derived from a scalar potential φ that satisfies Poisson's equation:

$$-\nabla \cdot (\varepsilon \nabla \varphi) = \sum_k q_k n_k. \tag{2}$$

Dirichlet boundary conditions are applied at both electrodes: the emitter ($\varphi = V_{DC}$) and the collector ($\varphi = 0$). Zero-flux boundary conditions are set for positive ions and neutrals at the positive electrode, while negative species are absorbed. At the grounded electrode, zero-flux conditions apply to neutrals and negative ions, and positive ions are absorbed. Secondary electron emission at the collector is modeled with a constant coefficient γ.

The equations are discretized using the finite volume method. The drift term is discretized using a first-order upwind scheme. MATLAB was used for the simulations, with vectorization to improve performance [5]. The time integration was performed using MATLAB's ode15s solver, suitable for stiff problems. This approach differs from the explicit Runge-Kutta scheme used in [1].

Quasi-2D Approach

In this study, a quasi-2D method is used for numerical simulations, reducing computational cost by restricting the computational domain between two iso-lines of a harmonic electric field. This method assumes all quantities are constant along the thickness direction, converting a problem that typically requires 2D simulation into a variable-section 1D simulation. This approach is particularly effective for wire-to-wire geometries, where the exact solution for the harmonic electric potential is known. The electric potential $\varphi^\star$ is calculated as the sum of the variable-section 1D Poisson solution φ and the harmonic 2D electric potential φ_{exact}:

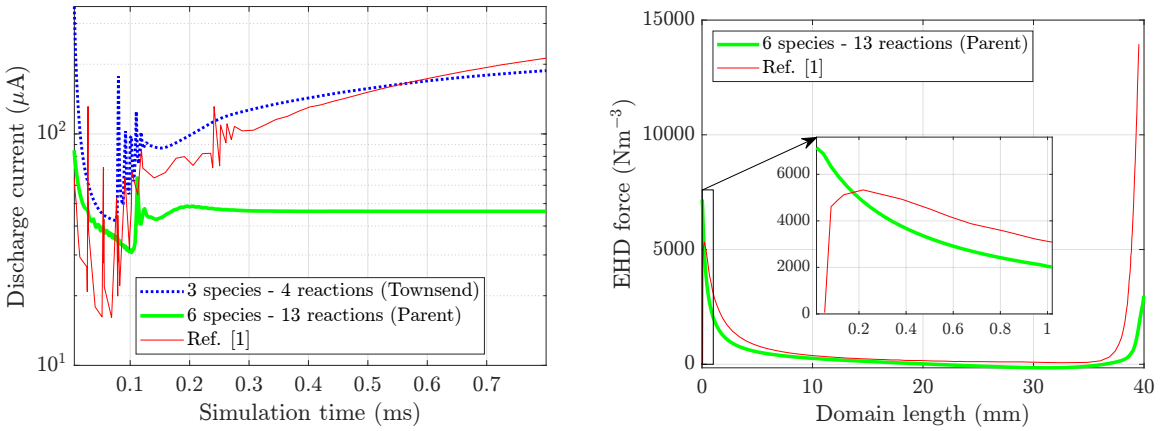
$$\varphi^\star(x) = \varphi(x) + \varphi_{exact}(x, 0). \tag{3}$$

The domain thickness $S(x)$ is determined by selecting two harmonic electric field lines. These lines are defined by the curve tangent to the electric field vector. In a 2D geometry, an electric field line $y(x)$ must satisfy the relation:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{E_y}{E_x}, \tag{4}$$

where E_x and E_y are the components of the harmonic electric field. For the wire-to-wire configuration, these components are given by [6]:

$$\begin{aligned} E_x(x,y) &= \frac{V_{DC}}{2\ln\left(\frac{A}{r_1}\right)} \left[\frac{2(x-\beta r_1)}{(x-\beta r_1)^2 + y^2} - \frac{2\beta(\beta x - r_1)}{(\beta x - r_1)^2 + (\beta y)^2} \right], \\ E_y(x,y) &= \frac{V_{DC}}{2\ln\left(\frac{A}{r_1}\right)} \left[\frac{2y}{(x-\beta r_1)^2 + y^2} - \frac{2\beta^2 y}{(\beta x - r_1)^2 + (\beta y)^2} \right]. \end{aligned} \tag{5}$$



(a) Discharge current.

(b) EHD force.

Figure 1: Comparison between the results obtained with different kinetic models. The simulation parameters are: $r_1 = 350\,\mu\text{m}$ (left emitter radius); $r_2 = 1\,\text{mm}$ (right collector radius); $R = 1\,\text{k}\Omega$ (external circuit resistance); $V_{DC} = 40\,\text{kV}$ (external circuit voltage generator); $d = 40\,\text{mm}$ (distance between the wires); $L = 16\,\text{cm}$ (axial length of the wires); $S_{max} = 5\,\text{mm}$ (maximum thickness of the computational domain); $\gamma = 1 \times 10^{-4}$ (secondary emission coefficient).

Here, r_1 is the emitter radius and A and β are geometrical parameters [6]. The first field line we considered is the horizontal line connecting the centers of the wires $y(x) = 0$. The second field line is determined by the angle θ, which intersects the first electrode at a specific point. The angle θ is found using a bisection method to match the desired maximum thickness S_{max} of the computational domain.

Results

Simulations were carried out using the same parameter set of [1] in order to compare the results we obtained with the ones in [1] (also referred to as Matéo-Vélez model results). In Fig. 1 the comparison between the obtained results is shown. Instead of computing the discharge current with the total current density, as in [7], it was computed using the generalized version of Sato's equation, as done in [8]. The discharge current is shown in Fig. 1a. It is possible to see that the time evolution of the discharge current yielded by the Townsend model and the Matéo-Vélez model is in good agreement. The current computed with the Parent model has a similar trend only in the first part of the simulated time period and then reaches a steady-state value that is significantly different from the one of the other models. In Fig. 1b the electro-hydro-dynamic force, computed as $f_{EHD} = \rho E$ is shown for the Matéo-Vélez and Parent model. When the Parent model is used, the force density is larger near the electrode with a smaller radius. The results obtained in [1] show a different situation, with a higher force density found near the electrode with a bigger radius. This highlights that the kinetic model has a significant impact on

the electro-hydro-dynamic force density results.

Conclusion

The impact of two different kinetic models on the computed electric current and electrohydrodynamic force density produced by a corona discharge in dry atmospheric-pressure air has been examined. The models included a 3-species, 4-reactions Townsend model and a 6-species, 13-reactions model from [3]. They were also compared with the Matéo-Vélez model in [1]. The results obtained with the Parent model for both current and force density were significantly different from the ones yielded by the other models. This highlights the significant influence of the chosen kinetic model on discharge behavior. Consequently, future research will aim to identify the essential set of reactions for ionic propulsion applications. Additionally, the quasi-2D approach used in this study enabled the analysis of a 2D geometry using a 1D simulation, leading to significant time savings. This method may be used in future studies for efficient preliminary evaluations of ionic-wind thruster performance.

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