FPTM 2.0: Fully relativistic bounce-averaged Fokker-Planck code for stellarators and tokamaks

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The radial profiles of power deposition and driven current during the Electron Cyclotron Resonance Heating (ECRH) of both tokamaks and stellarators are typically computed with ray-tracing codes suitably integrated into transport codes. E.g., for the case of stellarators the transport code NTSS [1] and the ray-tracing code TRAVIS [2] are used in conjunction. This kind of modeling is based on the assumption of Maxwellian plasmas.

On the other hand, in many fusion-relevant plasma scenarios kinetic effects may play a key role. E.g., the electron distribution function can significantly deviate from Maxwellian under highly focused launching conditions of the EC waves, and several kinetic codes simulating the evolution of RF-heated plasmas have been developed (see, for example, Ref. [3]).

However, most of these codes have been written explicitly for axisymmetric (tokamak) magnetic configurations. One exception is the code FPTM (Fokker-Planck for Toroidal Mirrors) [4], where several populations of trapped particles with different heating conditions are taken into account with a bounce-averaging procedure using a simplified stellarator magnetic configuration valid close to the axis. The code adopts a non-relativistic treatment for the collisions and a weakly-relativistic approximation for the RF heating. The code has been used for the description of on-axis ECRH and current drive (ECCD) in the W7-AS stellarator [5]. Here we report on the details of a completely new version of this code (FPTM 2.0), where a fully relativistic approach for both collisions and RF heating is used. A suitable interface for coupling FPTM 2.0 code with TRAVIS EC ray-tracing code is under development.

For the simulation of typical ECRH scenarios with central deposition profiles in stellarators, we can limit the analysis to the treatment of magnetic configurations where the toroidal mirror term dominates in the Fourier expansion of **B** (hence neglecting its radial and poloidal variations). To be more explicit, the magnetic configuration is approximated as that of a thin magnetic flux tube of length $2\pi R_0$, R_0 being the major radius of the torus, and N_r identical ripples in the toroidal direction, representing the N_r segments of the stellarator. The bounce-averaging procedure is applied in each segment.

Below, $\mathbf{u} = \mathbf{v}\gamma$ is the momentum per unit mass, with $\gamma = \sqrt{1+u^2/c^2}$ the relativistic factor, and $\theta = \arccos(u_\parallel/u)$ is the pitch-angle. The subscripts $\parallel$ and $\perp$ denote in general the components of the relevant vector parallel and perpendicular to the magnetic field, respectively. The toroidal angle in the magnetic ripple is indicated with $\varphi \in [-\pi/N_r, \pi/N_r]$, and the index 0 labels quantities evaluated at the minimum $B_{\min}$ of the magnetic field.

The bounce-averaging is defined as

$$\langle \ldots \rangle_B = \frac{1}{H_0} \int_{-\varphi^*}^{\varphi^*} \ldots \frac{\cos\theta_0}{\cos\theta} d\varphi \quad \text{with} \quad H_0 = \int_{-\pi/N_r}^{\pi/N_r} \frac{d\varphi}{b(\varphi)}, \tag{1}$$

where $\sin^2\theta = b(\varphi)\sin^2\theta_0$, with $b(\varphi) = B(\varphi)/B_{\min}$. For trapped particles $\theta_{0tp} < \theta_0 < \pi/N_r - \theta_{0tp}$, with the point of reflection, φ^*, defined by the condition $u_\parallel(\theta_0, \varphi^*) = 0$, i.e., $b(\varphi^*) = 1/\sin^2\theta_0$. Finally, the boundary between trapped and passing particles is given by $\theta_{0tp} = \arcsin(B_{\min}/B_{\max})^{1/2}$, with $B_{\max}$ the maximum value of B.

Let f_{e0}^n denote the bounce-averaged distribution function of the different electron populations ($n = \pm$ for passing particles circulating in opposite directions along the magnetic field, and $n = 1, \ldots, N_r$ for the particles trapped in the N_r toroidal mirrors). The relevant bounce times can be defined as

$$\tau_B^{\pm} = 2\pi R_0 \frac{\lambda_1(\theta_0)}{v|\cos\theta_0|}, \quad \tau_B^{1,\ldots,N_r} = \frac{2\pi R_0}{N_r} \frac{2\lambda_1(\theta_0)}{v|\cos\theta_0|}, \tag{2}$$

with $\lambda_1(\theta_0) = \langle 1 \rangle_B$.

The problem can then be formulated as a system of 2D Fokker-Planck equations,

$$\lambda_1 \frac{\partial f_{e0}^n}{\partial t} - \frac{e}{m_e} \langle E_\parallel \frac{\partial f_{e0}^n}{\partial u_\parallel} \rangle_B = \langle C_{\text{lin}}(f_{e0}^n) \rangle_B + \langle Q_{RF}(f_{e0}^n) \rangle_B - \langle K_{loss} f_{e0}^n \rangle_B + \langle S_{ext} \rangle_B, \tag{3}$$

for the $(2+N_r)$ distribution functions f_{e0}^n. Here, Q_{RF} is the RF heating term, while $E_\parallel$, K_{loss} and S_{ext} are possibly present terms of parallel electric field, sink and source, respectively.

$$C_{\text{lin}} \simeq C_D^{e/e}(f_e; f_{eM}) + C_I^{e/e}(f_{eM}; f_e) + \sum_i C_D^{e/i}(f_e; f_{iM}) \tag{4}$$

is the linearized collision operator. The index i denotes the ion populations, the subscript M a Maxwellian distribution, and C_D and C_I are the differential and integral parts of the operator, respectively, which are computed using a fully relativistic approach following Ref. [6].

The ion distribution functions are assumed to be Maxwellian with constant temperatures. In the term $C_I^{e/e}$ the electron distribution function is approximated as $f_e = f_{eM} + f_{e1}(u)\cos\theta$. This corresponds the so-called "truncated" collision operator [7], which conserves the number of particles and the parallel momentum, but not the energy. However, this approach is adequate since the density and temperature profiles are obtained from a transport code, where all terms (including ECRH) are computed for a Maxwellian distribution function. In this sense, the Fokker-Planck model provides information on the contribution to power deposition and current generation in the plasma resulting from the deviation from the Maxwellian.

Formally, the bounce averaged quasi-linear RF term can be written as

$$\langle Q_{RF}(f_{e0}^n)\rangle_B = \frac{1}{\tau_b^n}\frac{\partial}{\partial J^i}\left(\tau_b^n \langle D^n_{J^iJ^k}\rangle_B \frac{\partial f_{e0}^n}{\partial J^k}\right), \tag{5}$$

where J^i are invariants of the unperturbed motion, such as u and $\mu = m_e u_\perp^2/2B$ (magnetic moment). Here, a transformation in the appropriate coordinate system is implied.

The diffusion coefficients for u and the cross-diffusion coefficients can all be expressed in terms of the diffusion coefficient for the magnetic moment, which is computed in a form similar to Ref. [3],

$$\langle D^n_{\mu\mu}\rangle_B = \mathrm{const}\,\frac{|A_l^n|^2}{\tau_b^n |u_\parallel|}\sqrt{\frac{\pi}{\Delta W}}\exp\left[-\frac{1}{\Delta W}\left(\gamma - N_\parallel\frac{u_\parallel}{c} - \frac{l\omega_{ce}}{\omega}\right)^2\right], \tag{6}$$

where $N_\parallel$ is the parallel refractive index, and ω_c and ω are the electron gyrofrequency and the wave frequency, respectively. The factor $|A_l^n|^2$, with l the cyclotron harmonic number, depends on the wave polarization (it vanishes for trapped particle populations without direct heating), and ΔW denotes a finite width of the resonance layer, defined again similarly to Ref. [3]. The normalization constant is determined by the absorbed power computed with a Maxwellian distribution function.

For the solution of the system of Fokker-Planck equations (3) both the usual continuity of the distribution function and the conservation of particle fluxes across the boundary between passing and trapped particles are required. The latter condition couples the equations for all particle populations. Assuming that the electrons crossing the boundary can be trapped in any toroidal mirror with equal probability, this condition can be written as

$$\Gamma^+_{\theta_0}(\theta_{0tp}-\varepsilon)+\Gamma^-_{\theta_0}(\pi-\theta_{0tp}+\varepsilon) = \frac{1}{N_r}\sum_{n=1}^{N_r}\left[\Gamma^n_{\theta_0}(\theta_{0tp}+\varepsilon)+\Gamma^n_{\theta_0}(\pi-\theta_{0tp}-\varepsilon)\right]. \tag{7}$$

For simplicity, the width ε of the boundary between passing and trapped particles in Eq. (7) is assumed vanishingly small. Finally, the numerical solution of Eqs. (3) is found by means of a conservative alternating direction implicit scheme, where the integral part of the Coulomb operator is recalculated at each time step.

Here we report one example of application of the code. ECRH at the second harmonic O-mode with a minimum of B at the launching position is found to lead to very high power densities for the electrons trapped in the local mirror at the launching plane. The formation of a

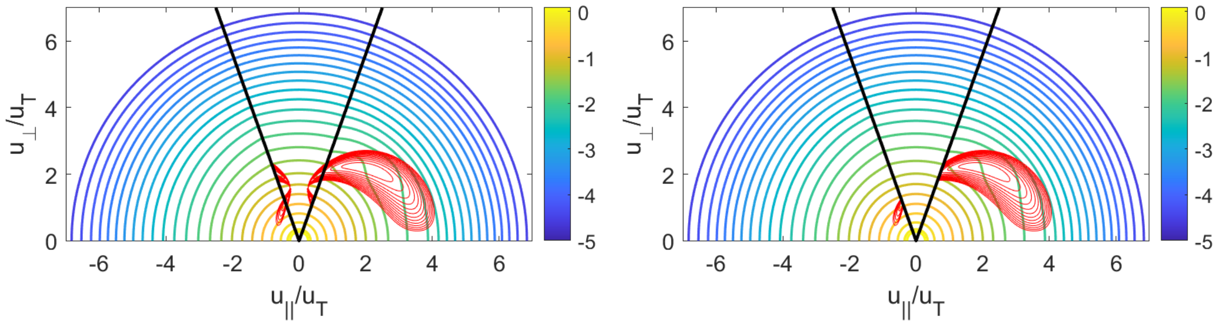

Figure 1: Contour levels of the bounce-averaged distribution function and the quasi-linear diffusion term (red lines) in $(u_{\|}, u_{\perp})$ space ($\mathbf{u}$ is normalized over $u_T = \sqrt{2T_e/m_e}$), using $B(\varphi) = B_0(1 - \delta\cos(\varphi))/(1 - \delta)$, with $B_0 = 2.50$ T and $\delta = 0.06$, with $N_r = 5$ segments along the torus. The plasma parameters are: $T_e = T_{i1} = T_{i2} = 5$ keV, $n_e = 0.75 \cdot 10^{20}$ m^{-3}, $n_{i1} = 0.675 \cdot 10^{20}$ m^{-3} (H^+), $n_{i2} = 0.0125 \cdot 10^{20}$ m^{-3} (C^{6+}) (for the given n_e, these ion densities correspond to an effective charge $Z_{eff} = 1.5$). Second harmonic ordinary waves at 140 GHz are launched at an angle of $\simeq 17°$ with respect to the normal to $\mathbf{B}$ ($N_{\|} = 0.25$, $N_{\perp} = 0.80$). The power density is 25 MW/m^3. Left: segment where trapped electrons are heated directly by the EC waves. Right: segment without direct heating of trapped electrons.

strong tail is found for this population of electrons. Fig. 1 shows the bounce-averaged electron distribution function as a function of the parallel and perpendicular momenta per unit mass, computed in the toroidal position of $B_{\min}$. A deviation from the initial Maxwellian distribution, though much smaller, can be observed in the other toroidal mirrors due to the collisional energy transfer.

References

[1] Yu. Turkin, C.D. Beidler, V. Erckmann, H.P. Laqua, H. Maaßberg, N.B. Marushchenko and W7-X Team, Proc. of the 34th EPS Conference on Plasma Physics (Warsaw, Poland, 2007), ECA Vol. 31F, P1.148 (2007)

[2] N.B. Marushchenko, Y. Turkin and H. Maassberg, Comp. Phys. Commun. **185** 165 (2014)

[3] E. Westerhof, A.G. Peeters and W.L. Schippers, *"RELAX : a computer code for the study of collisional and wave driven relaxation of the electron distribution function in toroidal geometry"*, Rijnhuizen report 92-211 (1992)

[4] N. Marushchenko, U. Gasparino, H. Maaßberg and M. Romé, Comp. Phys. Commun. **103**, 145 (1997)

[5] M. Romé, V. Erckmann, U. Gasparino, H.J. Hartfuß, G. Kühner, H. Maaßberg and N. Marushchenko, Plasma Phys. Control. Fusion **39**, 117 (1997)

[6] B.J. Braams and C.F.F. Karney, Phys. Fluids B **1**, 1355 (1989)

[7] C.F.F. Karney, Comp. Phys. Rep. **4**, 183 (1986)