Numerical and analytical calculations of orbit-wise synthetic spectra from two-step gamma-ray fusion reactions

A. Valentini[1], M. Nocente[2], J. Eriksson[3],
H. Järleblad[4], D. Moseev [5], B. C. G. Reman[6], M. Rud[1], A. Snicker [7], L. Stagner[8], M. Salewski[1]

[1] Department of Physics, Technical University of Denmark, 2800 Kgs. Lyngby, Denmark [2] Department of Physics, University of Milano-Bicocca, 20126 Milan, Italy [3] Department of Physics and Astronomy, Uppsala University, 75120 Uppsala, Sweden [4] Department of Appl. Math. & Computer Science, Technical University of Denmark, 2800 Kgs. Lyngby, Denmark [5] Max-Planck-Institut fur Plasmaphysik, Wendelsteinstr. 1, Greifswald 17491, Germany [6] Laboratory for Plasma Physics LPP-ERM/KMS, B-1000 Brussels, Belgium [7] Department of Applied Physics, Aalto University, FI-00076 Aalto, Finland [8] General Atomics, P.O. Box 85608, San Diego, California 92186-5609, USA

Introduction. In present and future large-scale fusion devices, the presence of energetic particles – due to external heating and fusion reactions – is important to investigate, given how these non-Maxwellian populations affect the plasma confinement [1, 2]. One method to measure the distribution function of fast ions is to infer it from the products generated via fusion reactions. In most cases, the cross sections of such reactions become relevant at hundreds of keV's of energy, well outside the Maxwellian velocity-range of the bulk plasma. These fast reactants leave traces in the nuclear radiation [3] via energy and momentum conservation: it is pivotal to model the nuclear emission and detection so to understand the measured signals correctly. In this work, we expand on previous calculations made in 2D velocity space [4] by evaluating synthetic spectra in the so-called *orbit space*, where velocity and configuration coordinates are mixed due to the orbital motion of the particles at these energy ranges [5]: transit/bounce timescales are much faster than collisional slowing-down [6], such that we can represent distribution functions in terms of orbit coordinates [7]. Here we consider two-step gamma-ray fusion reactions, where an excited nucleus is first generated and only then de-excites producing a photon that is observed along a specific detector line-of-sight. This reactions are relevant when measuring fusion-born alphas, and especially so at energies close to or higher than the nominal birth energy [8].

Methods. We consider a test magnetic equilibrium on JET (Culham, UK), in which toroidal symmetry holds, such that every valid orbit traced by the fast ions is represented by a coordinate triplet of energy E, maximum major radius R_{m} along the midplane and pitch at that position $p_{\mathrm{m}} = (v_{\parallel}/v)_{\mathrm{m}}$. By means of the Orbit Weight Computational Framework (OWCF) [9], the guiding-center relativistic Lagrangian equations-of-motion are integrated for a few orbital turns, after which the full 3D trajectory is superimposed – in the poloidal plane – onto the line-of-sight geometry of a given detector. At (R, z) points where an orbit is observed, a Monte-Carlo calculation is performed to calculate the energy spectrum of the product, given an (E, p) pair for the fast reactant – where E is conserved and p varies along the orbit. We choose the fast ion to be an alpha particle α and the observed product to be a 4.44 MeV photon emitted by an excited carbon-12 $^{12}C^{*}$ via

$$\alpha + {}^{9}Be \rightarrow n + {}^{12}C^{*} \rightarrow n + {}^{12}C + \gamma \ . \tag{1}$$

Numerically, the computation can be very demanding since several degrees of freedom have to be sampled sufficiently to obtain the correct signal. Additionally, unless a clever sampling of the excited-product velocity is made (which involves re-normalization of the single events), a fraction $f \gtrsim 0.5$ of the events can be non-physical and for this reason rejected. In the approximation of stationary target, which is especially solid in the case of 3.5 MeV alphas reacting with a 5 to 10keV bulk, we attempt to calculate the excited product distribution analytically and check against the Monte Carlo method. We can do this by calculating the analytical energy spectrum [10, 11] of $^{12}C^{*}$ via

$$\frac{\mathrm{d}^2 n}{\mathrm{d}E_{\mathrm{C}}\,\mathrm{d}t} = \frac{n_\alpha n_{\mathrm{Be}}}{\pi} v_\alpha \frac{\mathrm{d}\sigma(v_\alpha, p_\alpha, \Gamma_\alpha, p_{\mathrm{C}})}{\mathrm{d}\cos\theta_{\mathrm{lab}}} \left| \frac{\mathrm{d}\Gamma_\alpha}{\mathrm{d}E_{\mathrm{C}}} \right| \ ; \tag{2}$$

the double-differential spectrum is obtained by varying the pitch values $p_{\mathrm{C}} \in [-1, 1]$ for the excited product. n stands for density and v for velocity; the gyro-angles Γ are the degree of freedom which we can avoid sampling, along with two angles that parameterize the emission of the product in spherical coordinates. It follows that, for every alpha cold ring (v_α, p_α), we obtain a 2D distribution of $(E_{\mathrm{C}}, p_{\mathrm{C}})$, from which a relativistic Doppler-shift spectrum is calculated for the gamma-ray emitted by the excited carbon-12 towards the synthetic diagnostic.

Results. Three different calculations are made and compared for similar conditions. By this, we mean that specific orbits are selected such that a minimal overlap is made with the diagnostic line-of-sight: this way we expect the orbital and geometrical effects to be minimized – such that simpler codes can be used as well. An example is shown in figure 1 for a banana-type orbit. In the first case, the orbit-wise calculation is made with the full detailed geometry in account

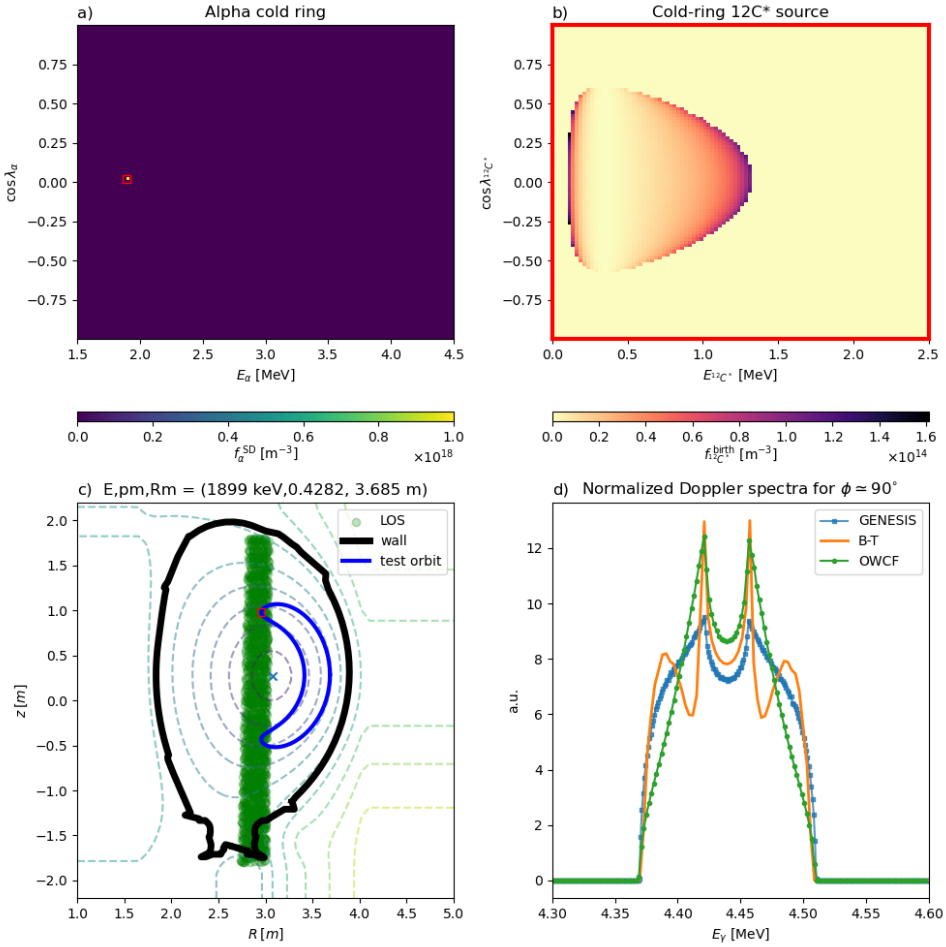

Figure 1: (Top:) a single alpha particle cold ring is chosen (a), from which the double-differential spectrum for the excited product is calculated (b). (Bottom:) a single orbit is selected (c) to simplify the comparison (d) between the resulting gamma-ray spectra from OWCF, GENESIS and the analytical code (BT).

using OWCF. The density and temperature of the Beryllium-9 have a typical peaked profile with $n_{\mathrm{Be}} = 10^{18}$ m^{-3} and $T_{\mathrm{Be}} = 5$ keV in the core of the plasma. In the second case, the GENESIS code [12, 13] is applied on a single alpha particle cold-ring purposefully placed at the (R, z) point of interest; density and temperature of the bulk are also taken into account here. In the third case, similarly, the beam-target code (BT) is used to estimate the spectra entirely analytically by also placing a cold-ring in the same location – this time neglecting the temperature (and so, velocity) of the second reactant.

Conclusions. Future work will be done to verify the source(s) of discrepancy between the codes, which can be due to the simplifications we introduced in the latter two models (GENESIS and BT), or to an insufficient number of events sampled while using the first model (OWCF). A full orbit-wise benchmark is in the plans to make sure the two-step reaction are properly implemented into OWCF as well.

Acknowledgements. This work has been carried out within the framework of the EUROfusion Consortium, funded by the European Union via the Euratom Research and Training Programme (Grant Agreement No 101052200 — EUROfusion). Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union or the European Commission. Neither the European Union nor the European Commission can be held responsible for them.

References.

[1] Heidbrink W W 2008 *Physics of Plasmas* **15** 055501

[2] Mazzi S *et al* 2022 *Nature Physics* **18** 776–782

[3] Moseev D *et al* 2018 *Reviews of Modern Plasma Physics* **2** 7

[4] Salewski M *et al* 2015 *Nuclear Fusion* **55** 093029

[5] Rud M *et al* 2024 *Nuclear Fusion* **64** 036007

[6] White R B 2001 *Theory of Toroidally Confined Plasmas (second Edition)* (Imperial College Press)

[7] Järleblad H *et al* 2024 *Nuclear Fusion* **64** 026015

[8] Salewski M *et al* 2018 *Nuclear Fusion* **58** 096019

[9] Järleblad H *et al* 2024 *Computer Physics Communications* **294** 108930

[10] Valentini A *et al* A model for analytical calculations of synthetic neutron energy spectra from beam-target reactions . Submitted to Nuclear Fusion,

[11] Valentini A *et al* Relativistic calculations of neutron and gamma-ray spectra from beam-target reactions in magnetized plasmas . Submitted to Review of Scientific Instruments,

[12] Nocente M 2011 *Neutron and gamma-ray emission spectroscopy as fast ion diagnostics in fusion plasmas* Ph.D. thesis University of Milano Bicocca

[13] Nocente M *et al* 2012 *Nuclear Fusion* **52** 063009