Towards an integral model resolving ion species effects and the ExB resonance in the kinetic plasma response to magnetic perturbations

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The next-generation tokamak experiment ITER will be operated in high confinement mode (H-mode). This operational mode is desirable in terms of plasma performance, however, it is also inflicted with instabilities occurring at the plasma edge. Edge localized modes (ELMs) threaten the lifetime of plasma-facing components in future fusion devices. Therefore, controlling ELMs is a requirement to ensure the safe operation of ITER. One method to suppress ELMs is the application of small resonant magnetic perturbations (RMPs) generated by additional field coils. However, the plasma initially responds to the RMPs with parallel electron-driven shielding currents and prevents the transition to ELM suppression. Nevertheless, specific access windows still allow for said transition through bifurcation, meaning, that certain perturbation modes penetrate the plasma and corrugate the electromagnetic fields. In particular, the perturbation mode that is resonant with the equilibrium magnetic field on the pedestal top [1] is assumed to be decisive in achieving ELM suppression. The specific details of this method are yet to be completely explored. Therefore, to reliably achieve RMP ELM suppression in any present or future device, we want to fully understand the relevant physics.

In this report, we motivate an integral approach to the linear plasma response to magnetic perturbations within kinetic theory and present the progress made in its development. The integral approach advances an existing kinetic response model in a straight cylinder with identical ends that is based on a finite Larmor radius expansion (FLRE) [2]. By accurately resolving ion Larmor radius effects, the integral approach potentially explains the recent observation in ASDEX Upgrade and DIII-D that RMP ELM suppression is lost when a deuterium plasma is mixed with hydrogen or helium [3]. Furthermore, the existing model does not resolve the rotational resonance associated with the electric field reversal point, commonly termed ExB or gyrocenter resonance [2]. However, modeling done with the FLRE model indicated that in three of four studied ASDEX Upgrade discharges this resonance, instead of the electron fluid resonance, might be responsible for mode penetration and the transition to ELM suppression [4]. Hence, to fully understand the process of bifurcation we need to accurately resolve the gyrocenter resonance in addition to the fluid resonance.

The linear plasma response to externally produced magnetic perturbations is given by the electromagnetic field perturbations mediated by the plasma. To arrive at a set of equations solvable for the field perturbations, the charge and current density perturbations in Maxwel's equations have to be expressed in terms of the field perturbations. An integral constitutive relation casts the equations into a set of integro-differential equations to solve. However, for the first prototype, we focus on studying electrostatic modes, that is, we want to solve Poisson's equation. In terms of integral kernels, Poisson's equation is given by

$$-\Delta\delta\Phi(r) = 4\pi\int \mathrm{d}r'\, K^{\rho\Phi}(r,r')\delta\Phi(r') + 4\pi\int \mathrm{d}r'\, K^{\rho B}(r,r')\delta B^r(r'), \tag{1}$$

where the right-hand side is the charge density perturbation $\delta\rho$ written in terms of the relevant field perturbations, and r is the cylinder radius. The kernels are determined by the velocity moments of the distribution function perturbation of the linearized plasma kinetic equation. Similar to the FLRE model [2], the plasma kinetic equation is solved by means of a Fourier series expansion in the angle variables of a set of action-angle coordinates. The angles $\theta = (\phi, \vartheta_g, z_g)$ are the gyrophase, and the poloidal angle and the toroidal coordinate of the gyrocenter, corresponding to the modes $\mathbf{m} = (m_\phi, m, k_z)$, where m_ϕ is the cyclotron number, m is the poloidal mode number and $k_z = n/R_0$ with the toroidal mode number n and the major radius R_0. The Fourier harmonics of the perturbation fields are written as

$$\Phi_{\mathbf{m}} = \frac{1}{(2\pi)^3}\int \mathrm{d}^3\theta\, \Phi(r)\mathrm{e}^{-\mathrm{i}m_\phi\phi + \mathrm{i}m\rho^\vartheta + \mathrm{i}k_z\rho^z}, \tag{2}$$

where ρ^ϑ and ρ^z are the poloidal and toroidal components of the Larmor vector. For the integral approach, the radial dependency is represented as a Fourier transformation,

$$\delta\Phi(r) = \int \mathrm{d}k_r\, \mathrm{e}^{\mathrm{i}rk_r}\Phi_{k_r}. \tag{3}$$

This choice allows for a mostly analytical derivation.

The kernel connecting the charge density perturbation and the electrostatic potential perturbation in the case of Krook collisions is determined for species σ as

$$K^{\rho\Phi}_{\sigma,mn}(k'_r,k_r) = \frac{1}{2^3\pi^2}\int_0^\infty \mathrm{d}r_g\, \frac{1}{\lambda_{D\sigma}^2}\mathrm{e}^{\mathrm{i}(k_r-k'_r)r_g}\left\{-\exp\left(-\frac{(\rho_{TL\sigma})^2}{2}(k_r-k'_r)^2\right) + \mathrm{e}^{-b_+}\frac{k_s\rho_{TL\sigma}}{k_\parallel\sqrt{2}}\right.$$
$$\left.\times\left[A_1^\sigma I_0(b_\times)Z(z_0) + A_2^\sigma\Big(Z(z_0)I_0(b_\times)(1+b_+ + z_0^2) + I_{-1}(b_\times)b_\times + z_0 I_0(b_\times)\Big)\right]\right\}, \tag{4}$$

where r_g is the gyrocenter radius, $\lambda_{D\sigma}$ is the Debye length, I_n is the modified Bessel function of the first kind, $\rho_{TL\sigma} = v_{T\sigma}/\omega_{c\sigma}$ is the thermal Larmor radius with the thermal velocity $v_{T\sigma} =$

$\sqrt{T_\sigma/m_\sigma}$ and the cyclotron frequency $\omega_{c\sigma}$. Further,

$$b_+ = \frac{k_\perp^2 + k_\perp'^2}{2}(\rho_{TL\sigma})^2, \qquad b_\times = k_\perp k_\perp'(\rho_{TL\sigma})^2, \qquad k_\perp^2 = k_s^2 + k_r^2, \tag{5}$$

$$k_s = \frac{1}{r_g}(h_z m - h_\vartheta k_z), \qquad k_\parallel = mh^\vartheta + k_z h^z, \qquad k_z = \frac{n}{R_0}, \tag{6}$$

and

$$A_1^\sigma = \frac{1}{n_\sigma}\frac{\partial n_\sigma}{\partial r_g} - \frac{e_\sigma E_r}{T_\sigma} - \frac{3}{2T_\sigma}\frac{\partial T_\sigma}{\partial r_g}, \qquad A_2^\sigma = \frac{1}{T_\sigma}\frac{\partial T_\sigma}{\partial r_g} \tag{7}$$

are thermodynamic forces depending on particle density n_σ, temperature T_σ and the equilibrium radial electric field E_r. The covariant (contravariant) components h_ϑ (h^ϑ) and h_z (h^z) are of the equilibrium magnetic field direction $\mathbf{h} = \mathbf{B}_0/B_0$. Moreover, $Z(z_0)$ is the plasma dispersion function depending on the argument

$$z_0 = -\frac{\omega_E - \omega - \mathrm{i}\nu_\sigma}{k_\parallel\sqrt{2}v_{T\sigma}} \tag{8}$$

with the ExB rotation frequency $\omega_E = -k_s c E_r/B_0$, the perturbation frequency ω and the collision frequency ν_σ.

In the definition of the kernel (4), we see that the second term in the curly brackets becomes important close to the resonance surface where $k_\parallel \to 0$. Further, the Bessel functions incorporate the gyro motion, whereas the plasma dispersion function accounts for Landau damping. The first term in the curly bracket provides Debye shielding in the case of a homogeneous plasma with static perturbations and no collisions. Moreover, note that the non-trivial part of the kernel is driven by the thermodynamic forces A_1^σ and A_2^σ and thus by the gradients of the plasma parameters and the radial electric field.

Concerning isotope effects, the kernel depends on the species mass in the thermal Larmor radius $\rho_{TL\sigma} \propto \sqrt{m_\sigma}$ and $z_0 \propto \sqrt{m_\sigma}$. The latter is also affected by the gyrocenter resonance. In particular, considering static perturbations without collisions, the parameter vanishes at the gyrocenter resonance. At this point, the plasma dispersion function is solely imaginary.

The ideal method to solve Poisson's equation numerically is yet unknown. The issue is the convergence of the Fourier spectrum. Solving the problem directly in Fourier space is not practical as a large number of modes is required. One solution is to consider the Fourier transformation of the radial dependency as a choice of function representation and as such it can be transformed to a new basis. Using bra-ket notation, the integral kernels can be written as

$$K_{l'l} = \langle l'|\hat{K}|l\rangle = \int_{-\infty}^{\infty} \mathrm{d}k_r \int_{-\infty}^{\infty} \mathrm{d}k_r' \, \langle l'|k_r'\rangle\langle k_r|l\rangle K(k_r', k_r) = \int_{-\infty}^{\infty} \mathrm{d}k_r \int_{-\infty}^{\infty} \mathrm{d}k_r' \, \varphi_{lk_r'}^* \varphi_{l'k_r} K(k_r', k_r) \tag{9}$$

where $\varphi_{lk_r} = \langle k_r|l\rangle$ is the Fourier transformed of the new basis function $\varphi_l(r)$. In total, this introduces four more integrals of which two, the Fourier transformation of the new basis functions, can be done analytically for a suitable choice of new basis functions, e.g. hat basis functions. Finally, the remaining three integrals (over k_r, k_r' and r_g) need to be solved numerically to determine $K_{l'l}$. In the function space φ_l, we can then write

$$\delta\rho(r) = \sum_{l=1}^{N_l} \varphi_l(r)\delta\rho_l, \qquad \sum_{l'=1}^{N_l} (\Delta_{ll'} + 4\pi K_{ll'}^{\rho\Phi})\delta\Phi_{l'} = -4\pi \sum_{l'=1}^{N_l} K_{ll'}^{\rho B}\delta B_{l'}^r, \tag{10}$$

where Poisson's equation is given in weak form with $\Delta_{ll'} = \int \mathrm{d}r \varphi_l^* \Delta \varphi_{l'}$.

Another possible basis is a localizer-Fourier series approach, that is

$$\delta\Phi(r) = \Theta(r) \sum_{k_r=-k_r^c}^{k_r^c} \Phi_{k_r} \mathrm{e}^{\mathrm{i}k_r r}, \tag{11}$$

where $\Theta(r)$ is a smooth function localized around the resonant surface generously including the resonant layer and k_r^c is a cut-off wavenumber. This results principally in a faster convergence of the Fourier series, reducing the required number of modes. Possibly, adding a polynomial contribution to the Fourier series expansion allows for dealing with non-periodic contributions. This approach can be either proceeded by basis transformation (9) with the new basis $\varphi_l(r) = \Theta(r)\mathrm{e}^{\mathrm{i}lr}$, or by starting from (2) and re-deriving the integral kernels.

Which approach is ultimately better suited is to be determined in future work. In any case, to finally determine the electromagnetic field perturbations, the integral representations of $\delta\rho$ and $\delta j_{\|}$ are plugged into Maxwell's equations. This results in a set of one-dimensional integro-differential equations in the radial variable. Then, combining the linear model with the quasilinear transport code QL-Balance [2] allows for determining the evolution of plasma parameters subjected to RMPs. This framework is then suitable for studying the open questions concerning the effects of isotopes and the significance of the ExB resonance on the plasma response, and the bifurcation of RMPs to fully understand the mechanisms behind ELM suppression.

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