Notes on orbit squeezing and density variation in neoclassical theory

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Abstract

Some basic tenets of the neoclassical theory are discussed here. It is known that plasma flow velocity $\boldsymbol{V}$ is incompressible in neoclassical theory. Consequently, the component of $\boldsymbol{V}$ parallel to the magnetic field $\boldsymbol{B}$, i.e., $V_{\parallel}$ is not a flux function. The poloidal variation of $V_{\parallel}$ contributes to a term that is of the same order as the drift speed, and the perturbed distribution is driven by the anisotropic $\left(v^2 - 3v_{\parallel}^2\right)$ and not $\left(v^2 + v_{\parallel}^2\right)$ dependence in the drift kinetic equation. consistent with the moment of the parallel viscous forces. Here, v is the particle speed, and $v_{\parallel}$ is the particle speed parallel to $\boldsymbol{B}$. The poloidal variations of the equilibrium density and electrostatic potential are neglected in neoclassical theory. However, when the *real* flow speed is sonic, these variations cannot be neglected. Note that the speed involved is the real flow speed not just the $\boldsymbol{E} \times \boldsymbol{B}$ speed. Here, $\boldsymbol{E}$ is the electric field. What happens often is that both the $\boldsymbol{E} \times \boldsymbol{B}$ and the diamagnetic flow speeds are large but the real flow speed remains sub-sonic in the high confinement mode (H-mode) in tokamaks. Thus, the poloidal variations of the density and the potential can be neglected there. The density and potential variations in the sonic flow has already been addressed. Thus, the results in the original orbit squeezing theory and in the subsequent extensions remain valid as they are.

I Introduction

In the edge region of the high confinement mode (H-mode) the radial scale length of the radial electric field is of the order of the banana width that is set up by the ion orbit loss in tokamaks [1,2]. Because the property of the radial electric field profile deviates from the conditions of the standard neoclassical theory [3-5], the neoclassical physics introduced by this novel profile has been studied and the results were reported decades ago in [6-11]. These results have been reviewed in [5]. A recent surprising development is that the dependence of the ion heat conductivity on the orbit squeezing factor S is actually converged to the one first derived three decades ago [6], which is also the scaling in the extension of the theory [7-11]! However, there are still discrepancies. Here, we show that these differences are results of the inconsistent and unrealistic assumptions on the plasma flows used by the authors of the new work. When plasma flows that are consistent with the tenets of neoclassical theory reviewed in [1-3] and realistic flow speeds are employed, these discrepancies should be resolved.

II Parallel Flows and Density Variation in Tokamaks

Plasma flow patterns in doubly periodic toroidal plasmas that are valid for arbitrary symmetry properties have been addressed in [5]. Here, we will just present the patterns for axisymmetric tokamaks. We will use the case of the incompressible flow to illustrate the physics. The results for the compressible flow will be outlined without deriving them in details.

From the parallel (to the equilibrium magnetic field $\boldsymbol{B}$) components of the momentum and heat flux balance equations, we conclude that plasma density N and temperature T are flux functions, for example χ , the poloidal flux divided by 2π, i.e., $N = N(\chi)$ and $T = T(\chi)$. However, the parallel plasma flows are not flux functions. They must satisfy the density and energy conservation laws.

To proceed further, we present the magnetic coordinates to be used. The standard representation of $\boldsymbol{B}$ in tokamaks is

$$\boldsymbol{B} = I\,\nabla\zeta + \nabla\zeta \times \nabla\chi\ , \tag{1}$$

where I is the poloidal current outside the flux surface divided by $c/2$, c is the speed of light, and ζ is the toroidal angle. We will denote the unit vector in the direction of $\boldsymbol{B}$ as $\boldsymbol{n}$.

We decompose the flow vectors into the perpendicular and parallel components to obtain

$$\boldsymbol{V} = \boldsymbol{V}_{\perp} + V_{||}\boldsymbol{n}\ , \tag{2}$$

and

$$\boldsymbol{q} = \boldsymbol{q}_{\perp} + q_{||}\boldsymbol{n}\ , \tag{3}$$

where $\boldsymbol{V}$ and $\boldsymbol{q}$ are respectively the mass and heat flows, and the subscripts denote the parallel $||$ and perpendicular $\perp$ components to $\boldsymbol{B}$. The lowest order (in the gyro-radius ordering) momentum and heat flux equations yield the perpendicular components of the flow velocities

$$\boldsymbol{V}_{\perp} = c\,\frac{\boldsymbol{B}\times\nabla\Phi}{B^2} + c\,\frac{\boldsymbol{B}\times\nabla p}{NeB^2}\ , \tag{4}$$

and

$$\boldsymbol{q}_{\perp} = c\,\frac{5p}{2}\frac{\boldsymbol{B}\times\nabla T}{eB^2}\ , \tag{5}$$

where $\Phi = \Phi(\chi)$ is the equilibrium electrostatic potential, p is the plasma pressure which is a flux function, and e is the charge of the species. Utilizing the facts that flow velocities lie on the flux surface, implied by Eqs. (2)-(5), the density and energy conservation laws reduce to

$$\boldsymbol{\nabla} \cdot \boldsymbol{V} = 0 , \tag{6}$$

and

$$\boldsymbol{\nabla} \cdot \boldsymbol{q} = 0 . \tag{7}$$

Using Eqs. (1) – (5), and solving Eqs. (6) and (7) for parallel flow speeds yield

$$V_{||} = BV^{\theta} + \frac{IcT}{eB}\left(\frac{p'}{p} + \frac{e\Phi'}{T}\right) , \tag{8}$$

and

$$q_{||} = Bq^{\theta} + \frac{5}{2}\, p\, \frac{IcT}{eB}\, \frac{T'}{T} , \tag{9}$$

where the superscript θ indicates the contravariant component of the flow velocities, and θ is the poloidal angle. The contravariant components are flux functions. The parallel flow speeds in Eqs. (8) and (9) are obviously not flux functions. The θ dependence of the flows is important to neoclassical theory. It is the θ dependence that eventually leads to the anisotropic $\left(v^2 - 3v_{\parallel}^2\right)$ dependence in the driving terms of the drift kinetic equation to be solved for the perturbed distribution in neoclassical theory. Indeed, the solution presented in [3-5] are identical in this regard. The only difference is in the treatment of the energy dependence in the temperature gradient term. In [3], a variational parameter y is introduced, while in [4] and [5], an 8-moment method is employed. Thus, the difference is a matter of taste in a sense. However, assuming that parallel flows are flux functions is not consistent with the density and energy conservation laws and is not justified. Indeed, it is this poloidal variations of the parallel flows that lead to the Pfirsch-Schluter current and transport fluxes in the Pfirsch-Schluter regime. Our o orbit squeezing theory and subsequent extensions are absolutely consistent in this regard.

In a sonic rotating tokamak plasma, plasma density and the electrostatic potential are not flux functions. From the density conservation law

$$\nabla \cdot (N\boldsymbol{V}) = 0, \tag{10}$$

we conclude that the flow velocity $\boldsymbol{V}$ has the following form

$$\boldsymbol{V} = \boldsymbol{B}\frac{K(\chi)}{N} - \left(c\Phi' + \frac{cp'}{Ne}\right) R^2 \nabla\zeta , \tag{11}$$

where $K(\chi) = N\boldsymbol{V} \cdot \boldsymbol{\nabla}\theta / \boldsymbol{B} \cdot \boldsymbol{\nabla}\theta$ is a flux function, and R is the major radius. It is clear that when the $\boldsymbol{E} \times \boldsymbol{B}$ and the diamagnetic flow speeds are large but almost cancel each other, the real flow velocity $\boldsymbol{V}$ can still be subsonic. The significant diamagnetic flow should be a result of the turbulence suppression in the H-mode layer [1,2]. In that case, the density and potential variations are negligible as demonstrated in [8]. The diamagnetic flow has to be incorporated in any realistic theory, a point emphasized repeatedly in [5]. Thus, the poloidal variations of the density and potential calculated by assuming that $\boldsymbol{E} \times \boldsymbol{B}$ flow dominates are not relevant to experiments.

III Concluding Remarks

The dependence on the orbit squeezing factor S in ion heat conductivity in the orbit squeezing theory has finally converged to the one first presented in [6] and in the subsequent extensions in [7-11] decades ago. Here, we point out the basic tenets of the neoclassical theory that should be adhered to in the development of the theory to resolve the remaining discrepancies. First, the parallel flow components cannot be flux functions to satisfy density and energy conservation laws. This leads to the anisotropic $\left(v^2 - 3v_\parallel^2\right)$ dependence in the drift kinetic equation to be solved for the perturbed distribution. Secondly, the poloidal density and potential variations depend on the *real* flow speeds not the $\boldsymbol{E} \times \boldsymbol{B}$ flow only. In the edge region of the H-mode, the real flow speed is not large. The poloidal variations of the density and potential for sonic rotating plasmas have already been addressed in [8]. We conclude that all the decades-old results in [6-11] remain valid as they are.

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