A unified approach to plasma shape control with coil current allocation in magnetic confinement fusion

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Shape control is crucial in plasma magnetic confinement due to its direct impact on fusion efficiency. Traditional controllers focus on minimizing plasma shape errors but often overlook other critical aspects, such as maintaining coil currents within operational limits. Neglecting these factors can lead to confinement loss or an inability to respond during emergency shutdowns. Previous studies have addressed these challenges using control allocation techniques, exploiting coil redundancy relative to the number of controlled shape outputs to optimize current requests without altering the plasma shape. This work introduces a novel unified approach by formulating the plasma shape control and the coil current allocation as a nonlinear optimal control problem, which is solved using a gradient descent algorithm. The effectiveness of this approach is evaluated through simulations conducted on the Tokamak à Configuration Variable (TCV) model and data.

1 Introduction

In magnetic confinement fusion, controlling the plasma shape is essential for maintaining the stability and efficiency of the fusion process. Therefore, developing robust and efficient control strategies for plasma shape is a critical area of research in pursuing sustainable fusion energy.

Traditional control methods primarily focus on minimizing shape errors and deviations from the desired plasma boundary [1, 2, 3]. A significant limitation of these approaches is their tendency to overlook other crucial aspects, particularly the behavior of coil currents. Poloidal field coils are essential for plasma shaping, and neglecting their operational limits can compromise the integrity of the confinement system. Furthermore, in case of emergency shutdown scenarios, the ability to respond promptly and effectively is paramount, requiring that coil currents remain within safe operational boundaries, as in runaway electron beam control [7].

Previous research has addressed some of these challenges by employing control allocation techniques [4, 5, 6]. These techniques leverage the redundancy inherent in the coil system, given that the number of coils typically exceeds the number of controlled plasma shape outputs. This redundancy allows the distribution of current demands among multiple coils, optimizing the overall current requests while maintaining the desired plasma shape.

This paper introduces a novel approach that integrates plasma shape control and coil current allocation into a unified nonlinear optimal control framework [8]. This method is distinguished by its formulation of the control problem as a nonlinear optimization task, defined by a cost function that simultaneously considers plasma shape errors and poloidal coil currents. Each objective within the cost function is weighted to reflect its relative importance, allowing for a balanced optimization that prioritizes shape accuracy and current management.

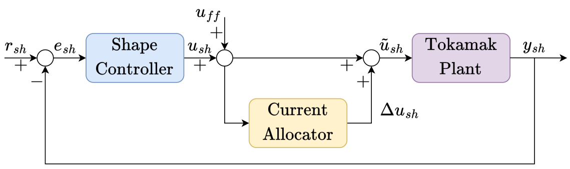

Fig. 1: State-of-the-art control scheme.

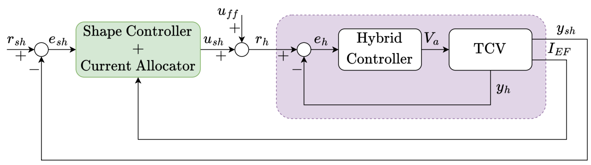

Fig. 2: Proposed control scheme.

2 State-of-the-art

The state-of-the-art approach in plasma shape control typically involves decoupling the problem into two distinct components: a shape controller and a coil current allocator [1, 2], as illustrated in Fig. 1. The shape controller's primary objective is to manage and correct the plasma shape errors. This goal is achieved by modifying the poloidal coil currents to adjust the magnetic field and poloidal flux at specific points that identify the desired plasma's Last Closed Flux Surface. While the shape controller handles the plasma shape, the coil current allocator focuses on the behavior of the coil currents. The allocator exploits, if present, the system redundancy, which provides the opportunity to distribute the current demands among the available actuators in an optimized manner without significantly modifying the plant output, that is, the plasma shape.

3 Proposed control scheme

This work introduces a novel approach integrating plasma shape control and coil current allocation into a unified framework. This integration is achieved by formulating the control objectives into a comprehensive cost function, which is minimized using a gradient descent algorithm.

Figure 2 shows a simplified version of the magnetic control systems of the TCV tokamak. The central plant TCV describes the dynamics of the plasma and coils. This model is obtained by linearization of the Grad– Shafranov equation, an elliptic PDE that describes the magnetohydrodynamic equilibrium of an axisymmetric tokamak plasma, coupled with the evolution of currents in the plasma and in the active circuits and passive structures of the tokamak [9]. The inner control loop is handled by the so-called *hybrid controller*, which computes the voltages V_a to be applied to the PF coils to track the references r_h. It takes care of the vertical and radial stabilization of the plasma and controls the current flowing in the PF coils.

Define the matrix $P_{sh}^{\infty} \in \mathbb{R}^{\eta \times 24}$ that represents the static gain from the input u_{sh} to the shape output y_{sh}, and the matrix $P_{EF}^{\infty} \in \mathbb{R}^{16 \times 24}$ that represents the static gain from the input u_{sh} to the EF coil currents I_{EF}. Using these matrices, it is possible to derive the relationships $y_{sh}^{\infty} = P_{sh}^{\infty} u_{sh} + d$ and $I_{EF}^{\infty} = P_{EF}^{\infty} u_{sh} + d$, where d encapsulates all the nonlinearities and uncertainties not considered in the linearized model.

The fundamental idea behind our approach is to condense the dual aspects of shape control and current allocation into a single optimization problem. This is accomplished by defining a weighted cost function $J(e_{sh}^{\infty}, I_{EF}^{\infty})$ that encapsulates all control objectives. To solve the optimization problem, we employ a gradient descent algorithm. The dynamic of the unified controller is described by

$$\dot{u}_{sh}(t) = -\rho \left(\frac{\partial J(e_{sh}, I_{EF})}{\partial u_{sh}} \right)^T ,$$

where $\rho > 0$ is the step size that regulates the convergence rate.

This integrated approach offers several advantages over the traditional decoupled method. Combining shape control and current allocation into a single cost function allows for balancing competing objectives and improves performance by considering their interdependence. This unified framework simplifies the design process and reduces computational effort, making it more efficient. In addition, the integrated approach provides the flexibility to introduce additional control objectives.

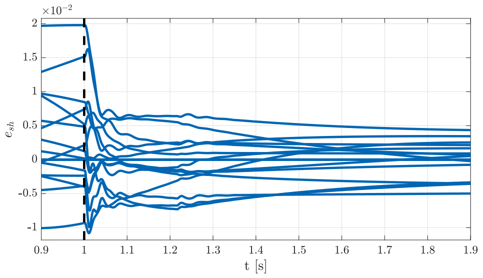

Fig. 3: Shape errors in shape control case.

Fig. 5: TCV shot #78732 at 1.9 s in shape control case.

Fig. 4: EF coil currents in shape control case.

4 Numerical simulations

This section corroborates the proposed solution through numerical simulations using the Matlab EQuilibrium Toolbox (MEQ). The following simulations consider the flat-top phase from 1.0 s to 1.9 s of TCV pulse $\#78732$. The corresponding plasma equilibrium is a negative-triangularity, lower single null configuration, and its shape is defined by 7 contour points, one X-point, and 2 strike points. Therefore, the size of the shape output is $\eta = 11$ because it contains the difference between the poloidal flux at the contour and strike points and the one at the X-point and the radial and vertical components of the magnetic field measured at the X-point.

The first simulation addresses the problem of minimizing the squared weighted norm of the shape errors. Specifically, this objective is achieved by defining the cost function as $J(e_{sh}^{\infty}) = \frac{1}{2}\, e_{sh}^{\infty\,T} W_{sh} e_{sh}^{\infty}$, where $W_{sh} \in \mathbb{R}^{\eta\times\eta}$, $W_{sh} = W_{sh}' \succ 0$ assigns weights to the different shape errors.

Figure 3 presents the plasma shape errors over time and highlights how the proposed shape controller design significantly reduces errors in less than 100 ms, demonstrating its efficiency. Figure 5 offers a snapshot of the plasma shape at 1.9 s. This figure compares the desired plasma shape (dashed black), the shape achieved using only the feedforward coil currents (orange), and the shape obtained using the proposed control scheme (blue). Feedforward currents are pre-programmed based on the nominal plant model and do not account for uncertainties, contrary to the presented feedback scheme, which achieves an almost perfect match to the desired shape. Figure 4 illustrates the coil current traces that, in this case, are not included in the controller's cost function.

The second simulation aims to minimize the squared weighted norm of the shape errors while also avoiding saturation of the EF coil currents and keeping them sufficiently far from zero (similar to [6]). In this case, the cost function is modified to $J(e_{sh}^{\infty}, I_{EF}^{\infty}) = \frac{1}{2}\, e_{sh}^{\infty\,T} W_{sh} e_{sh}^{\infty} + \sum_{i=1}^{16} \xi_i(I_{EF,i}^{\infty})$, with the functions $\xi_i(I_{EF,i}^{\infty})$ defined as

$$\xi_i(I_{EF,i}^{\infty}) = \begin{cases} \dfrac{1}{2}\,\alpha_i(I_{EF,i}^{\infty} - \delta_i)^2 & \text{if } |I_{EF,i}^{\infty}| > \delta_i\,, \\ \dfrac{1}{2}\,\beta_i(I_{EF,i}^{\infty} - \gamma_i)^2 & \text{if } |I_{EF,i}^{\infty}| < \gamma_i\,, \\ 0 & \text{otherwise}\,, \end{cases}$$

where $\alpha_i, \beta_i \geq 0$ for $i = 1, \dots, 16$ are the weights associated with the two desired regions, and $\gamma_i, \delta_i \geq 0$ for $i = 1, \dots, 16$ are the bounds defining these regions.