Investigation of the Shannon entropy of atomic states in dense plasma

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Introduction. Understanding the behaviour of dense plasma [1-11], a state of matter with high particle densities and strong interactions, is crucial in both theoretical and applied physics. In this connection, it is interesting to study the entropy of atomic states in dense plasma. Concept of information entropy, introduced by Claude Shannon [12]. Shannon's information entropy measures the uncertainty or disorder in a system, making it an essential tool for quantifying the complexity of atomic states in dense plasma environments. This article explores Shannon's information entropy in the context of atomic states in dense plasma, focusing on the effects of interaction potentials, quantum non-locality, electronic correlation, and ionic screening [13].

Quantum non-locality, a key concept in quantum mechanics, describes how particles can instantly influence each other's states, regardless of the distance between them. In dense plasma, this non-locality greatly affects the interaction potential, changing the distribution and entropy of atomic states. Additionally, electronic correlation, which goes beyond the mean-field approximation to describe electron interactions, adds more complexity and disorder to the entropy landscape.

Ionic screening, another important factor, changes the interaction potential due to the presence of other ions in the plasma. This screening reduces the long-range Coulomb interactions, reshaping the potential landscape and the entropy characteristics of atomic states. By accounting for ionic screening, we gain a better understanding of how localized interactions dominate in high-density environments, impacting the overall entropy.

Theory and methods. The effective potential was proposed in Ref. [13]. It was obtained based on the Coulomb potential and dielectric function, which takes into account quantum non-locality and electronic correlation.

The Coulomb potential in Fourier form is as follows:

$$\tilde{\varphi}_{\alpha\beta}(k) = \frac{4\pi Z_\alpha Z_\beta e^2}{k^2}. \quad (1)$$

The effective potential in Fourier form is given by formula [13]

$$\tilde{\Phi}_{\alpha\beta}(k) = \frac{4\pi Z_\alpha Z_\beta e^2\left(1+\lambda_{ee}^2 k_i^2\right)}{\lambda_{ee}^2 k^4+\left(1+\lambda_{ee}^2 k_i^2\right)k^2+k_D^2} \quad (2)$$

Using the inverse Fourier transform, the potential is determined by the following formula:

$$\Phi_{\alpha\beta}(r) = \frac{Z_\alpha Z_\beta e^2}{Cr}\left((1-\lambda_{ee}^2 B^2)\,exp(-Br) - (1-\lambda_{ee}^2 A^2)\,exp(-Ar)\right), \quad (3)$$

where $C = \sqrt{(1+k_i^2\lambda_{ee}^2)^2 - 4k_D^2\lambda_{ee}^2}$, $A^2 = \frac{1+k_i^2\lambda_{ee}^2+C}{2\lambda_{ee}^2}$, $B^2 = \frac{1+k_i^2\lambda_{ee}^2-C}{2\lambda_{ee}^2}$.

For $4k_D^2\lambda_{ee}^2 > (1+k_i^2\lambda_{ee}^2)^2$ the potential (3) takes the following form:

$$\Phi_{\alpha\beta}(r) = \frac{Z_\alpha Z_\beta e^2 d_{\alpha\beta}}{\xi r\sqrt{\left(\frac{2k_D}{\lambda_{ee}\xi^2}\right)^2 - 1}}\,sin\left(\sqrt{\frac{k_D}{\lambda_{ee}}}\,sin(\tfrac{\omega}{2})r + \theta_{\alpha\beta}\right)exp\left[-r\sqrt{\frac{k_D}{\lambda_{ee}}}\,cos(\tfrac{\omega}{2})\right], \quad (4)$$

here $d_{\alpha\beta} = \sqrt{a_{\alpha\beta}^2 + b_{\alpha\beta}^2}$, $\theta_{\alpha\beta} = arctan\left(\frac{b_{\alpha\beta}}{a_{\alpha\beta}}\right)$, $\omega = arctan\left(\sqrt{\left(\frac{2k_D}{\lambda_{ee}\xi^2}\right)^2 - 1}\right)$, $a_{\alpha\beta} = \frac{1}{2\lambda_{ee}^2} - \xi^2$, $b_{\alpha\beta} = \xi^2\sqrt{\left(\frac{2k_D}{\lambda_{ee}\xi^2}\right)^2 - 1}$.

The Shannon entropy linked to the probability distribution, expressed as $\rho(r) = |\psi(r)|^2$ in position space, can be described by the equation [14]:

$$S_\rho = -\int \rho(\boldsymbol{r})\,ln\,\rho(\boldsymbol{r})d\boldsymbol{r}^3 = -\int|\psi(\boldsymbol{r})|^2\,ln\,|\psi(\boldsymbol{r})|^2 d\boldsymbol{r}^3, \quad (4)$$

where $\psi(r)$ is the atomic wave function in position space *r*. For one-electron systems with the wave function $\psi(r)[= R_{nl}(r)Y_{lm}(\Omega)]$, the Shannon entropy S can be divided into radial and angular components:

$$S_\rho = S(R_{nl}) + S(Y_{lm}) \quad (5)$$

here $S(R_{nl})$ and $S(Y_{lm})$ are the radial and angular parts of the Shannon entropy [14], respectively.

Results. In Figures 1-2, the electron-ion interaction potential (3) is presented. Figure 1 shows various density parameter values while keeping the degeneracy parameter constant, and Figure 2, conversely, shows various degeneracy parameter values while keeping the density parameter constant. From the graphs, it can be concluded that the absolute values of the interaction potentials decrease as the density parameter decreases and the degeneracy parameter increases. The interaction potential, as described by equation (3), approaches infinity at short distances and sharply decreases in absolute value as the distance increases.