Algebraic and numerical studies on the roles of momentum conservation and self-adjointness in moment-based neoclassical particle fluxes

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Introduction

In the analysis of neoclassical transport in tokamak plasmas, the moment method has achieved great success. The moment approach, which utilizes ordered moment equations obtained by integrating the drift-kinetic equations multiplied by velocities, has enabled neoclassical calculations with fairly high accuracy and speed [1]. Its numerical implementations have become prevalent for quantitative experimental analyses and have been incorporated into integrated transport models, even though methods for directly solving the drift-kinetic equation numerically have been spreading in recent years. In the moment method, the collision operator is velocity-integrated to obtain the matrix elements, which are the essential components of the friction coefficients and viscosity coefficients.

Linearized collision operators typically used in the drift kinetic equation are model operators that approximate the nonlinear Landau collision operator, but cannot capture all the features of the Landau operator due to approximation. Various linearized collision operators have been proposed, including the one that ensures self-adjointness of the operator and another that maintains the friction-flow relations derived from the exact linearized collision operator, for example.

The choice of model collision operator in the drift kinetic equation influences the moment method through these matrix elements. However, there was no clear guidance on what problems, or for what code, to use a model that preserves what properties of collision operators, like self-adjointness or momentum conservation. To elucidate the basis for choosing an appropriate model operator, which derives the matrix elements used to express friction forces, we investigate the roles of momentum conservation and self-adjointness of the collision operator in the independence of the neoclassical particle flux and the particle poloidal flow from the radial electric field E_r in an axisymmetric system [2]. Hirshman analytically confirmed this fact based on the assumption that the viscous forces are negligibly smaller than the friction forces [1], but it has not been clear whether the same conclusion can be reached when no assumptions on the viscous forces are imposed. This investigation to elucidate it is conducted theoretically, algebraically, and numerically within the moment method framework.

Momentum conservation and self-adjointness

Let the symbol ℓ_{ij}^{ab} be the friction coefficient between species a and b. The subscripts i, j correspond to the velocity moment; for example, ℓ_{12}^{ab} denotes the friction coefficient multiplied by the heat flow ($j = 2$) in the expression for the particle friction force ($i = 1$). If the collision operator ensures momentum conservation,

$$\sum_a \ell_{1j}^{ab} = 0 \tag{1}$$

holds for $j = 1, 2$. If it possesses the self-adjoint property, it ensures the symmetry of the friction coefficients:

$$\ell_{ij}^{ab} = \ell_{ji}^{ba}. \tag{2}$$

The combination of momentum conservation and self-adjointness leads to

$$\sum_b \ell_{i1}^{ab} = 0 \tag{3}$$

for $i = 1, 2$.

In the process of deriving the equations certifying ambipolarity and the independence of the particle flux and the poloidal flow from E_r, whether momentum conservation or self-adjointness is necessary, or both are needed can be determined by whether or not these above relationships are used.

Ambipolarity and the independence of the particle flux and the poloidal flow from E_r

The primary force balances in the momentum and heat flux equations parallel to the magnetic field occur between the viscous forces and the friction forces. The friction-flow relations express the particle and heat friction forces as the combination of the particle and heat flows and the viscous forces are the functions of the poloidal flows. The parallel particle and heat flows can then be described in the matrix form consisting of the viscous and friction coefficients in terms of the diamagnetic flows and the parallel electric field after awkward linear algebraic calculations. Here, the diamagnetic particle flow consists of the pressure gradient and the electrostatic potential gradient (i.e., $\sim E_r$) and the diamagnetic heat flow is a function of the temperature gradient. Once the parallel flows can be expressed as a function of such thermodynamic forces, we then investigate whether the aforementioned relationships were used when deriving the matrix coefficients that appear in the equations for constructing the particle flux or the poloidal particle and heat flows.

Since inverting a matrix of any size is extremely complicated, the cofactor expansion as a method of calculating the inverse of the matrix was applied to the matrix elements we were focusing on, that is, those associated with the particle flux and E_r. As a result, it was found that while only momentum conservation is required to maintain ambipolarity, both momentum conservation and self-adjointness are necessary to maintain the independence of the particle flux and the poloidal flows from E_r [2]. This is the first time that these relationships have been mathematically proven to include finite viscosity.

In the case of Pfirsch-Schlüter particle flux and classical particle flux without viscosities, it can be proven in a straightforward manner that only momentum conservation is required for ambipolarity.

Numerical results

Among several numerical implementations of the neoclassical moment method, Matrix Inversion (MI) [3, 4, 5] is used for numerical tests. CHARROT [6, 7], which implements MI inside as a neoclassical transport solver, is the code that was originally developed to convert experimentally measured carbon toroidal rotation to deuterium one, but also has the features of calculating neoclassical transport quantities. Figure 1 displays CHARROT results including the neoclassical particle fluxes for each species and their sum to check ambipolarity for a typical L-mode plasma consisting of electrons, deuteriums and carbons with $T_\mathrm{e} \neq T_\mathrm{i} = T_\mathrm{C}$ and the finite $\langle BE_\parallel \rangle$ of $\sim 0.13\,\mathrm{TV/m}$. In this simulation, the friction coefficients derived by the exact linearized collision operator were used, indicating that momentum conservation holds but self-adjointness is slightly violated. The results demonstrate exact ambipolarity at the level of numerical rounding error.

From the standpoint of implementing the matrix elements in a transport code or a neoclassical transport solver, the emphasis should not be placed on self-adjointness. Instead, it is preferable to use the model that reproduces the same friction–flow relations as those provided by the exact linearized collision operator. It means that self-adjointness has nothing to do with ambipolarity.

Also, it can be found that the electron particle flux is in magnitude larger than the deuterium and carbon ones. In tokamak plasmas, where a parallel electric field is present whenever the ohmic current flows, this field induces an inward Ware flux. This flux can be substantial, where electrons may have the same or larger particle flux compared to main ions and impurities, depending on the magnitude of the parallel electric field. Sometimes the significance of the electron particle flux is neglected when modeling a neoclassical impurity flux, but considering the Ware flux and the existence of the parallel electric field in a tokamak plasma, the neoclassical electron flux must be retained. Detailed results and discussion should be consulted in [2].

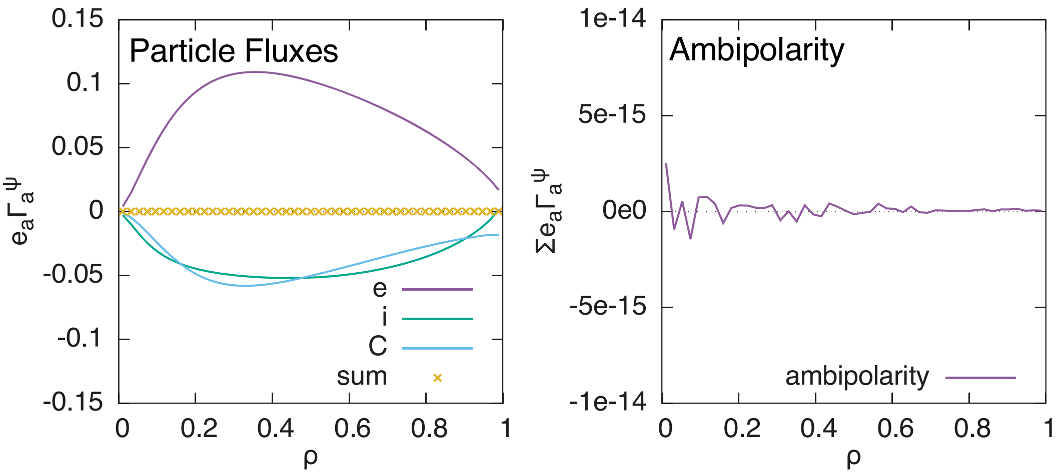

Figure 1: The neoclassical particle fluxes for each species (left) and the ambipolarity check (right).

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