Stochastic transport in Spherical Tokamaks

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Introduction

Many important issues to improve confinement time in tokamak devices are related to the development of plasma instabilities that characterize turbulence and consequently energy and particle transport. We can distinguish between electrostatic (ES) and electromagnetic (EM) plasma instabilities on the basis of the fact if we neglect or not magnetic field perturbations. Electromagnetic instabilities represent a more general and more complex class of perturbations with respect to the well-known electrostatic ones and can become significant, when ion/electron plasma pressure $P_k = P_i + P_e$ is finite or large compared with magnetic field pressure $P_B = B^2/(2\mu_0)$. Thus, it appears that the prediction of electromagnetic effects is particular important in Spherical Tokamaks (STs) that can operate at high $\beta = P_k/P_B$ values. STs are devices that offer several practical advantages such as radial compactness, potentially lower cost components with an improved plasma stability that allows operation at lower magnetic field compared to traditional tokamaks. In STs, the development of electromagnetic turbulence is often accompanied by a stabilization of electrostatic instabilities and shearing rate through which the turbulent cells are decorrelated. The picture that emerges is, however, extremely complex considering that electrostatic instabilities such as ITG become alfvénic at higher β with significant magnetic fluctuations, and microtering modes (MTMs) can become unstable. MTM instability is one of the most important and less understood electromagnetic instabilities in the tokamak context and its name is related to the mode's characteristic breaking of equilibrium magnetic field to generate localized magnetic islands. It is probably related to the development of a stochastic behavior of the magnetic field that represents a very efficient mechanism of heat transport at low values of magnetic field. Stochasticity represents one possible mechanism responsible of the strong electron heat transport, often observed in STs and that cannot be described by standard quasilinear models. Understanding and predict this kind of transport is extremely important by considering that fast ions produced by fusion process will primarily heat electrons in the future tokamak machines.

Stochastic models

A magnetic field can present a stochastic behaviour at the surface $q = m/n$ (with m and n poloidal and toroidal mode numbers) when the width of the localized resonant magnetic island,

w_i [1] exceeds the distance between rational surfaces, δr_{res}:

$$w_i = 4\sqrt{\frac{\delta B}{B_0}\frac{r}{n}\frac{R}{s}} \quad \delta r_{res} = \frac{1}{ndq/dr} \quad \rightarrow \quad \frac{\delta B}{B_0} > \frac{1}{16}\frac{r}{R}\frac{1}{q^2}\frac{1}{ns} \tag{1}$$

Thus, the stochasticity threshold is related to a critical mode amplitude that depends on n, safety factor $q \sim rB_0/RB_\theta$, and magnetic shear $s = r/q dq/dr$. The relative amplitude of magnetic fluctuations, $\delta B/B_0$, increases with β, and this quantity is often sufficiently large in STs to exceed the stochasticity threshold [2]. Although the problem of stochasticity is far from being completely understood, different reduced models have been developed to describe its correlated transport. The starting point, for many of them, is to associate a radial displacement δx to a magnetic fluctuation δB via the simple expression $\delta x/l_{||p} = \delta B/B_0$ in which $l_{||p}$ represents a characteristic length scale along the magnetic field direction z. Thus, a magnetic diffusivity can be defined as $D_{eff} = (\delta x)^2/\delta t = (\delta B/B_0)^2 l_{||p} v = D_m v$, where D_m is a coefficient providing a measure of the stochastic behavior of B and v is a characteristic velocity that depends on the adopted model. For example, in the Kadomsev Pogutse model [3] an expression for v is obtained assuming that when two stochastic perturbed magnetic field lines approach each other, an electron can jump from one to another line via collisional process. The model is collisional in the sense that the jump of a decorrelation length $l_{\perp c}$, required for an electron to be captured by another magnetic line, is entirely due to collisions. Another mechanism, discovered by Rechester an Rosenbluth (RR) [4, 5], more efficient than that one proposed in Ref.[3] highlights that two neighbouring magnetic field lines separated by a perpendicular distance x_d, at one location, become exponentially separated along the field direction z. Thus, because of the conservation of magnetic flux, it follows that a correlated region of flux in a stochastic field must expand and develop increasingly fine scale dendritic structures (of width $\delta_d \sim x_d e^{-z/l_K}$) in the perpendicular plane as z increases. Due to the formation of these structures, a very small collision is sufficient to cause an electron to move between uncorrelated field lines. Collisions, therefore, play the important role of seed of the process, in the RR- model. On the basis of these considerations it is possible to rewrite D_{eff} expression as follows (see Ref.s [4, 5, 6] for a derivation):

$$D_{eff} = D_m v = \left(\frac{\delta B}{B_0}\right)^2 \frac{l_{||p}}{l_{||c}} D_{||} \tag{2}$$

in which the velocity v is a function of a diffusivity $D_{||}$ and of a length scale $l_{||c}$, that represents a generalization of the Kolmogorov length scale l_k, because related also to the fix geometrical configuration via the magnetic shear value that changes along the radial direction.

Reduced models in JINTRAC and simulation results

In order to implement stochastic models in JINTRAC, different assumptions and approximations have been done. Nonlinear drift-kinetic theory of the stochastic turbulence suggests saturation occurs at amplitude $\delta B/B = \rho_e/L_T$ [7] (Drake hypothesis). Thermal conduction co-

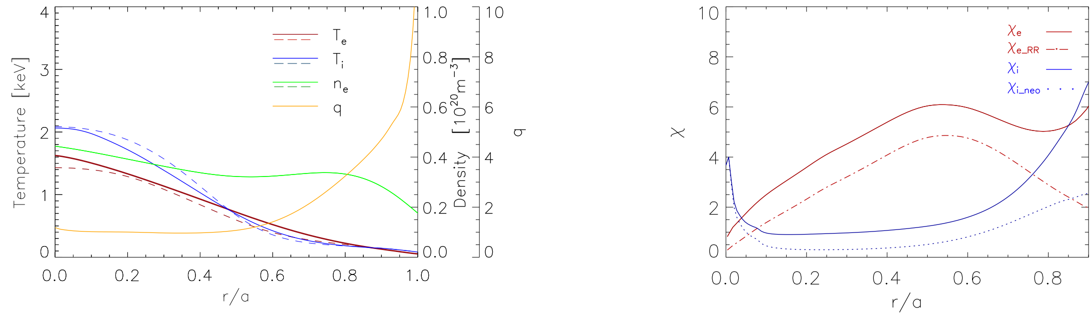

Figure 1: *(Left panel) Comparison between T_e, T_i, den exp. profiles (dash lines) and profiles obtained by using $RR_{\mathrm{c-less}}$ + BgB (continuous line). Density profiles are overlapped because n_e has been assumed interpretative. (Right panel) Transport diffusion coefficient profiles in $[\mathrm{m}^2/\mathrm{s}]$ units.*

efficients for electrons, derived from kinetic theory, have been considered equal to $D^e_{||} \sim \chi^e_{||} \sim v_e\lambda_{mfp}$. The length scale is assumed $l_{||p} \approx R$, because of the order of the tokamak machine. Moreover, the important length scale expression $l_{||c} \approx \pi R/\ln[\pi w_i/(2\delta r_{res})]$ comes from an analytical derivation obtained for a single toroidal n mode [4, 8]. The general formula $l_{||g}$ for a n spectrum is unknown, but it is possible to demonstrate that $l_{||g} < l_{||c}$. Thus, given a possible scenario, from the equilibrium conditions, by comparing the length scales λ_{mfp} and the $l_{||c}$, it is possible to select the suitable model to describe stochasticity. By using the mentioned approximations the following equations have been implemented in JINTRAC:

$$\chi_{eRR_{c-less}} \approx \left(\frac{\rho_e}{L_{T_e}}\right)^2 R v_e 2\sqrt{\frac{2}{\pi}}\left(1-\sqrt{\frac{r}{R}}\right) \qquad \chi_{eRR_{coll}} \approx \left(\frac{\rho_e}{L_{T_e}}\right)^2 R v_e \frac{\lambda_{mfp}}{l_{||c}} \tag{3}$$

for collisionless ($\lambda_{mfp} > l_{||c}$) and collisional ($\lambda_{mfp} < l_{||c}$) scenario. A hybrid version $\chi_{e_{Hy}} \approx (\chi^{-1}_{eRR_{c-less}} + \chi^{-1}_{eRR_{coll}})^{-1}$ for the possible scenario $\lambda_{mfp} \approx l_{||c}$ has been also implemented in the code. These reduced models have been coupled with TGLF (SAT1 and SAT2 saturation rules) to supplement the transport from this latter. In fact, while TGLF can be used in electrostatic or electromagnetic mode, in electromagnetic mode it has insufficient radial resolution to capture stochasticity derived from the development of instabilities such as MTMs. The resulting reduced transport model has been validated via a comparison between simulations and MAST/MAST-U experiments. Different selection criteria have been adopted to select the most suitable discharges. Shots that span in a certain range of collisional values have been considered because of the importance of the collisionality. Experimental discharges not affected by Magnetohydrodynamic (MHD) instabilities, whith small variations in time of quantities such as temperature and density have been selected. Thus, MAST shots 22664, 22769 and MAST-U shots 47003, 46978 have been analysed. The RR + TGLF(SAT1) has shown interesting results putting in evidence the important role that stochasticity can play in ST transport. A statistical analysis via a reduced $\bar{\chi}^2 = \chi^2/f_p$, with f_p number of degree of freedom, has shown a good agreement between experiments and simulation predicted profiles. This has been also confirmed by a low relative error $\Delta W_{e,i} = (W_{e,i_{sims}} - W_{e,i_{exp}})/W_{e,i_{exp}}$ on the stored energy $W_{e,i}$ (see Fig. 24 in Ref. [6]).

Results in [6] show a trend for which the importance of stochasticity with respect to the other ES/EM instabilities increases by increasing the $\lambda_{mfp}/l_{||c}$ ratio. This trend could be very useful in the assessment or qualification of the role of stochasticity in tokamak scenarios. We observe that the stochastic RR-model with the adopted approximation can be coupled with other models and improved in different ways. Here, we show preliminary results of RR + BgB model and we present a scan in the magnetic field in order to study the Drake hypothesis $\delta B/B = \rho_e/L_{Te}$ influence. For the BgB model the following expressions $\chi_{eBohm} = 0.5(T_e/B_0)aq^2|\nabla P_e|/P_e$, $\chi_{iBohm} = \chi_{eBohm}$ and $\chi_{gBohm} = \rho_i|\nabla P_i|/(q_i n_e B_0)$ (with q_i ion charge) has been adopted in JINTRAC. We have considered density, temperature profiles close to those of the analysed MAST case 22664 that presented large values of stochastic transport and for which the RR-collisionless model can be used. We selected major radius $R = 0.8m$, minor radius $a = 0.6m$, elongation $\kappa = 1.9$ and triangularity $\tau = 0.2$. By assuming predictive temperature profiles and interpretative density profiles a good agreement has been observed between simulations and experiments as shown in Fig. 1 in which initial electron T_e (red-dash lines), ion T_i (blue-dash lines) temperature, density n_e (green) profiles and T_e, T_i, n_e simulation profiles (continuous lines) are plotted. The yellow curves in the figure represents the safety factor. Diffusion coefficient profiles are shown on the right of Fig. 1. These profiles show a dominant electron stochastic transport in the middle of the radial box and lower ion transport. These results are in qualitative agreement with results obtained by running RR-model +TGLF(SAT1). Assuming this case as a reference, a scan in the magnetic field has been performed. Results shown in Fig. 2 put in evidence how the amplitude and shape of $X_{e_{RR}}$ decreases by increasing B. A detailed comparison between Bohm, gBohm and stochastic transport as a function of β values and other parameters will be presented in a future paper.

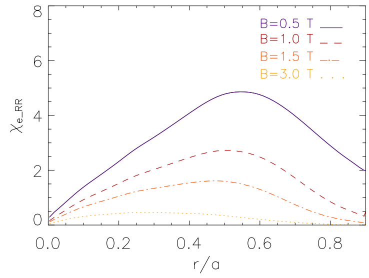

Figure 2: X_{eRR} *radial profiles in* $[m^2/s]$ *units, for different magnetic field values.*

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References

[1] Wesson J. *Tokamaks* 2004 third edition, Clarendon Press-Oxford
[2] Kaye S.M. et al. 2021 *Plasma Phys. Control. Fusion* **63** 123001
[3] Rosenbluth M.N. and Sagdeev R.Z. 1984 *Handbook of Plasma Physics (North-Holland, Amsterdam)*
[4] Rechester A.B. and Rosenbluth M.N. 1979 *Phys. Rev. Lett.* **40** 38
[5] Rechester A.B., Rosenbluth M.N.and White R.B. 1979 *Phys. Rev. Lett.* **42** 1247
[6] Palermo F. et al. 2025 *Plasma Phys. Control. Fusion* **67** 065004
[7] Drake J.F. and Lee Y.C. 1977 *Phys. Fluids* **20** 1341
[8] Zaslavsky V.G.M. and Chirikov B.V. 1972 *Sov. Phys. Usp.* **14** 549