Topology as a cornerstone feature of the magnetic fields

Ilan Roth

University of California, Space Sciences, Berkeley, CA , 94720, USA

The structure of magnetized plasma, its stability, evolution and modifications form a topic of major importance in Laboratory plasma, Solar physics, Planetary sciences, Astronomy and beyond. The consensus regarding the generation of the large-scale magnetic field converges on dynamo processes due to the movements of electrical charges, which in planetary domain include conductive molten iron and nickel in planet's core. The directly observed magnetic solar activity is a product of dynamo starting deep in the solar convection zone with the ensuing flux emergence into the overlying solar atmosphere. The resulting structures are generally described as distorted monopole loops with a possible twist and curvature, extending into large distances from the formation site.
We present here a different topological prototype of magnetized plasma which is consistent with Maxwell equations and observed configurations, while clearly distinguishable from the distorted magnetized plasma loops. The new structures take the forms of knots, facilitating an implementation of Braid/Knot Theory. The observed solar magnetic braids allow us to construct topological characteristics of these structures, predict their coronal dynamics and their emergence into the solar wind as composite knots, while extending these classifications into other magnetized domains in lab and astrophysics.

We describe the various formations of knotted magnetic fields in free space and extend them into the MHD domain, leading to transformation of coronal braids into knots, which are related to the observed large scale solar flare ion flux interruptions, as well as small-scale magnetic switchback rotations and interpret the emitted jets on tiny scale in lab, medium scale in ionospheric curls and huge scale in protoplanetary Harbig-Haro objects as stable, topological magnetized features.

The Hopf fibration is a map h between hyperspheres $S^3=\{(a,b,c,d)|a^2+b^2+c^2+d^2=1\}$ and $S^2=\{(x,y,z)|x^2+y^2+z^2=1\}$ with $h(a,b,c,d)=[a^2+b^2-c^2-d^2, 2(ad+bc), 2(bd-ac)] \epsilon S^2$. The inverse-transform h^{-1} forms *fibers*: two points on S^2 map into linked circles; three latitudinal circles map into nested links on R^3 which is formed as stereographic projection from the S^3 north pole (Fig 1)

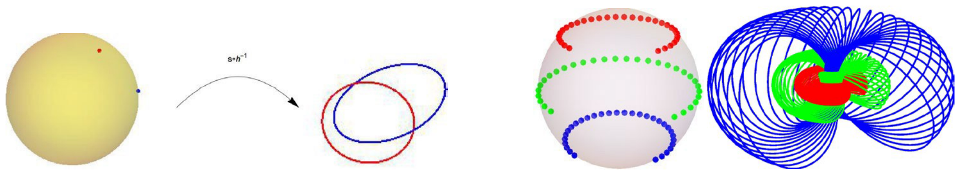

Figure 1. Hopf fibration: two points on S^2 mapped into two linked circles; three latitudinal circles into nested links.

Electromagnetic fields in vacuum are constructed through Hopf maps where the level curves of two complex scalar maps (θ,φ) projected on R^3 coincide with the electromagnetic vectors lines; E, B are the tangents of the Hopf fibers (Fig 2); while the *hopfion* fields deform in time, the topology and Poynting vector is preserved [1]. Full MHD plasma simulation of magnetic rings, on the other hand, result in Hopf-like configuration [2].

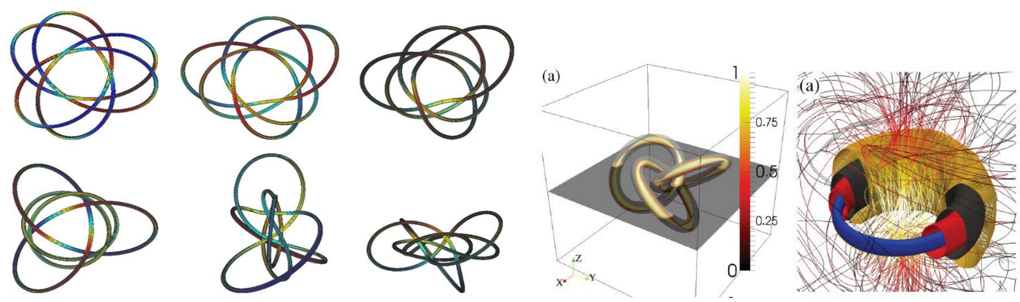

Figure 2. Left: B(upper), E(lower) fields for trefoil-(3,2) torus knots at T=0 and their successive temporal evolution.

Right: MHD evolution of three magnetic twisted rings, resulting in Hopf-like structure.

High resolution observations of the coronal field lines [3] reveal magnetic structures as braids - a disjoint collection of crossings among a number of strands attached to foot-points at two planes ("top" and "bottom"), with one-to-one correspondence between the foot-points (Figure 3, left), similarly to the Artin braids [4] - set of n disjoint strings in 3- space attached to two (horizontal) bars. The observed loops are undergoing moves preserving their topological equivalence as described by Artin braids group operations, which may be represented through algebraic expressions, while new loops light up, fade out or connect into more extended compact structures [5]. This equivalence allows us to predict several stable braid eigenmodes, which should be seen in high resolution observations. Braids and knots and their physical analogues at the solar corona form topologically related entities.

Magnetic fields in rectilinear configuration in the presence of opposite polarity fields are reconnecting into closed strings of magnetic islands, while a sheared bundle of braided fields

(Figure 3, left), deformed into more skewed form (Figure 3, center), may reconnect its successive strands through field *closure* into a new closed structure (Figure 3, right), i.e. magnetic knot. Mathematical knots and field lines of magnetized plasma in the MHD approximation are described as closed loops or bundles in three-dimensional (3D) space, transformed dynamically via continuous deformation of 3D upon itself, pushed smoothly in the surrounding viscous (plasma) fluid, respectively, without self-intersection. The physical "frozen in" condition for magnetic knots is equivalent to the knot "ambient isotopy" controlled by the three Reidemeister diagrammatic moves (e.g. [5]) as shown on Figure 4: R_1 - twist , R_2 -poke, R_3 - slide. R_j, j=1,2,3 moves reduce then the complicated topological problem to a simpler diagrammatic one, relating the changes in the observed projection to the relations between the crossings in the magnetic configuration. The invariance under Rj assures that any quantity which characterizes the magnetic field knot must preserve its value while undergoing the Rj transformations, becoming a topological invariant, which assigns uniqueness and stability to each knotted magnetic configuration. Non-equivalent magnetized knots are characterized by explicit helicity/winding invariants in the form of various polynomials (e.g., Alexander, Conway, Jones, Kauffman). Therefore, a transformation of any braid into a knot through the closure reconnection procedure indicates significant stability of the resulting magnetized structure.

The observed coronal magnetic loops are undergoing motions preserving their topological equivalence as described by Artin braids group operations, which may be represented by R_1, R_2 through algebraic expressions. This equivalence, together with Markov moves [7] allows us to predict stable braid eigenmodes, which should be seen in high resolution observations.

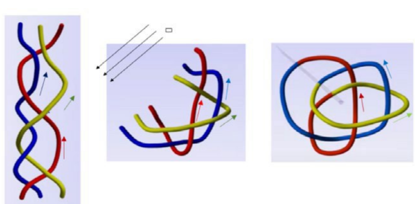

Figure 3. 3D non-laminar Braid (left) reconnection leading to formation of a knot (right)

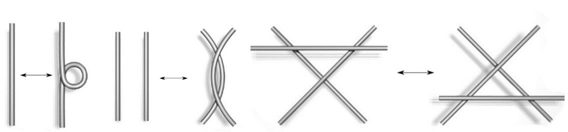

Figure 4. Reidemeister moves confirming invariance of a structure

The fast solar wind is accelerated largely by the local coronal pressure gradients and Poynting vector due to interchange reconnection of new emerging loops at the base of the corona with open magnetic field [8]. The braided reconnection events form clustered knots which emerge into the solar wind in the topological form of *knot sum*: two magnetic knots moving along close trajectories interact when the front of the trailing knot is merging with the tail of the leading knot. This operation happens numerous times between two adjacent knots, resulting in formation of elongated composite knots. The resulting switchbacks are the observed magnetic structures ejected into the solar wind. Example of knot sum of different prime knots embedded in the solar wind is shown on Figure 5: red arrows signify the field with reversals, blue lines (a, b, c) describe various encounters. This sequence of propagating knots, analogously to gun discharged bullets, forms due to topological invariants robust stable magnetic jetlets. Similar jetlets may operate in various laboratory and astrophysical features, like ionospheric curls and Herbig–Haro objects in the protostar outflows.

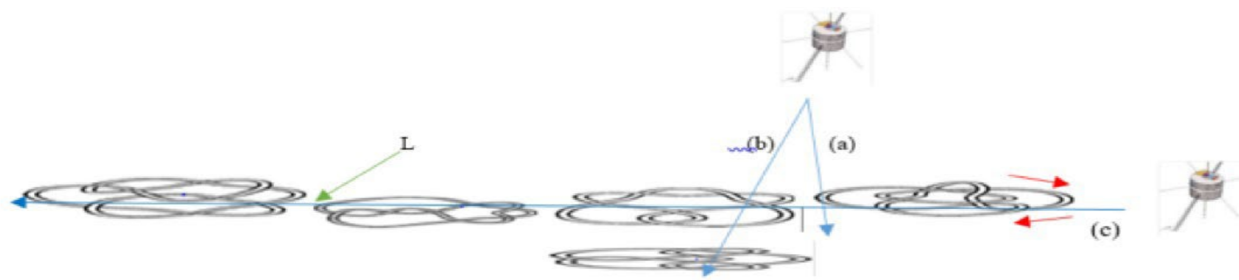

Figure 5. Encounters of a satellite with a collection of composite knots.

References

[1] Arrayás, M., 2017 Phys. Rep. 667, 1–61
[2] Smiet et al, 2015 Phys. Rev. Lett. 115, 095001
[3] Cirtain, J. W., et al, 2013, Nature, 493(7433), 501–503
[4] Artin, E, 1947, *Ann. of Math.*, 2, **48** (1): 101–126
[5] Thalmann, J. K., 2014, The Astrophysical Journal, 780(1), 102
[6] Adams, C. C. 2004, The knot book
[7] Markoff A., Mathematics, Moscow, 1936**, 1(43)**:1, 73–78
[8] Fisk, L. A., 1999, Journal of Geophysical Research: Space Physics, 104(A9), 19765–19772.