A fully coupled pressure-based method for the electron fluid model in Hall-effect thrusters

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The Hall-Effect Thruster is an electrostatic thruster that typically features an annular discharge channel. A radial magnetic field within the channel reduces electron mobility and induces the formation of a localized axial electric field near the channel exit. The ions are accelerated by this electric field, thereby generating thrust. A plasma model that effectively simulates this sequence of processes is the two-dimensional hybrid model, where ions and neutrals are represented as macroparticles, while electrons are modeled as a fluid. This numerical approach has been widely employed in Hall-Effect Thruster plasma simulations [1,2]. Conventionally, fully-2D hybrid models that utilize a magnetic field-aligned mesh to solve the electron fluid equations employ a segregated solution algorithm, where the electron momentum equation is approximated using the generalized Ohm's law under the assumption of inertialess electrons [3,4]. However, this approach requires multiple iterative computations to achieve convergence of the equation residuals. Additionally, a numerical method capable of solving the electron momentum equation in a more general form is necessary to account for electron inertia. This study presents the derivation of a fully coupled pressure-based method for solving the electron flow governing equations in a fully-2D method by formulating them into a single sparse matrix. Furthermore, a comparative analysis with simulations incorporating electron inertia is conducted to investigate the influence of electron inertia on plasma response.

In this study, the Hall-effect thruster (HET) plasma is simulated using a two-dimensional (r,z) hybrid model. Ions and neutral species are represented by variable-weight macroparticles, whose positions and velocities are tracked using the equations of motion. Electrons are modeled as a fluid, and their governing equations are as follows:

$$\sum_{f\in C} -\left(\mathbf{D}_f \cdot \nabla V_{T,f}^{t+\Delta t}\right)\cdot \mathbf{S}_f = -\sum_{f\in C}\langle \mathbf{J}_e\rangle_f^{t+\Delta t}\cdot \mathbf{S}_f - \sum_{f\in C}\left\langle \mathbf{D}\cdot \nabla V_T^{t'}\right\rangle_f \cdot \mathbf{S}_f - \sum_{f\in C}\mathbf{J}_{i,f}^{t+\Delta t}\cdot \mathbf{S}_f, \tag{1}$$

$$\left[\frac{\partial \mathbf{J}_e}{\partial t} + \nabla \cdot (\mathbf{u}_e \mathbf{J}_e)\right]\iota = \epsilon_0 \omega_{pe}^2 \left\{-\nabla V_T + \nabla T_e \left[1 - \ln\left(\frac{n_e}{n_{e0}}\right)\right]\right\} - \tilde{\boldsymbol{\nu}}_e \cdot \mathbf{J}_e - \boldsymbol{\nu}_e \cdot \mathbf{J}_h, \tag{2}$$

$$\frac{\partial}{\partial t}\left(\frac{3}{2} e n_e T_e\right) + \nabla \cdot \left(-\boldsymbol{\kappa}_e \cdot \nabla T_e - \frac{5}{2} T_e \mathbf{J}_e\right) = \mathbf{J}_e \cdot \left[\left(\boldsymbol{\nu}_e \cdot \mathbf{J}_e + \frac{1}{2}\nu_i \mathbf{J}_e + \boldsymbol{\nu}_e \cdot \mathbf{J}_h\right)\left(\epsilon_0 \omega_{pe}^2\right)^{-1}\right] - \mathbf{J}_e \cdot \left[\nabla T_e + T_e \nabla \ln\left(\frac{n_e}{n_{e0}}\right)\right] - \sum_s k_{es}^{\epsilon} n_e n_s - \sum_s \left(\frac{3 m_e}{m_s}\right) e n_e \nu_{es}(T_e - T_s). \tag{3}$$

The above equations correspond, in order, to the thermalized potential equation, the electron current conservation equation, and the electron energy equation. The thermalized potential equation is derived by substituting the Rhie-Chow interpolation of the electron current density at cell faces into the charge conservation equation $\nabla \cdot (\mathbf{J}_e + \mathbf{J}_i) = 0$. This interpolation establishes a $\mathbf{J}_e$-V_T coupling, analogous to the $\mathbf{u}$-p coupling commonly used in conventional computational fluid dynamics (CFD).

$$\mathbf{J}_{e,f}^{t+\Delta t} = \langle \mathbf{J}_e \rangle_f^{t+\Delta t} - \mathbf{D}_f \cdot \nabla V_{T,f}^{t+\Delta t} + \left\langle \mathbf{D} \cdot \nabla V_T^{t'} \right\rangle_f. \tag{4}$$

As a result, this approach is referred to as a pressure-based method. The dependent variables of Eqs. (1)–(3) are V_T, $\mathbf{J}_e$, and T_e respectively, and each of these variables is treated implicitly in its corresponding equation. After discretizing Eqs. (1)–(3), they are solved simultaneously within a sparse matrix framework. This is therefore called a coupled method, offering the advantage of enhanced numerical stability.

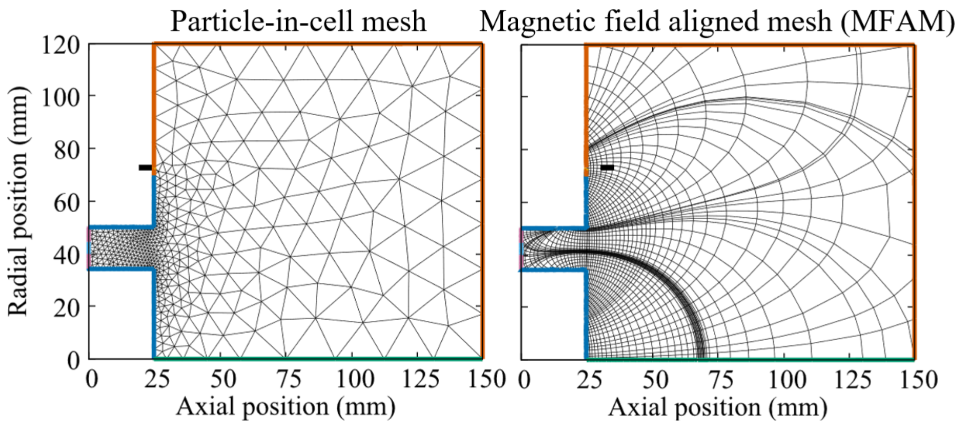

Figure 1. Particle-in-cell mesh (left) and magnetic field aligned mesh (right).

Figure 1 presents the computational meshes employed in this study. Using these meshes along with a time step of 20 ns and a discharge voltage of 300 V, the plasma parameters along the channel centerline calculated in this study are compared with those obtained from HYPICFLU [5] and HPHall-2 [6] in Fig. 2. The results derived in this study show reasonable agreement with those from HYPICFLU and HPHall-2. Therefore, the fully coupled pressure-based method is confirmed to exhibit comparable accuracy to existing approaches.

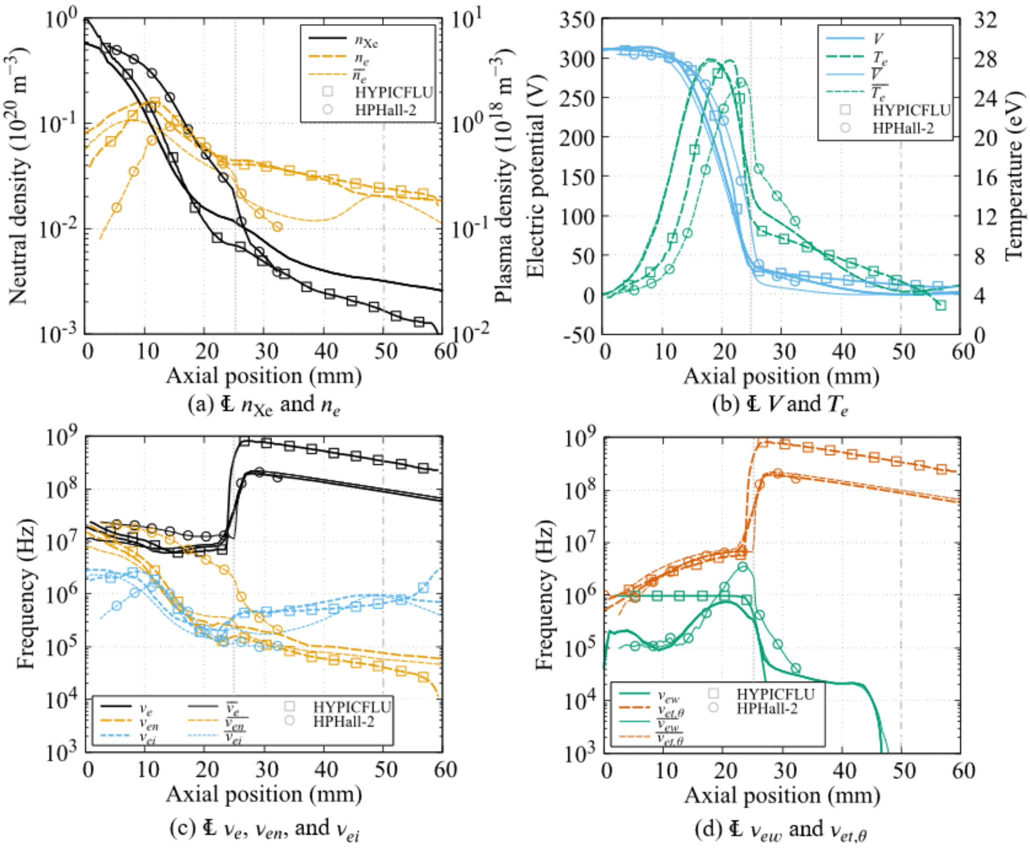

Figure 2. Comparison of channel centerline Xe density and plasma density (a), electric potential and electron temperature (b), collision frequencies (c), and (d).

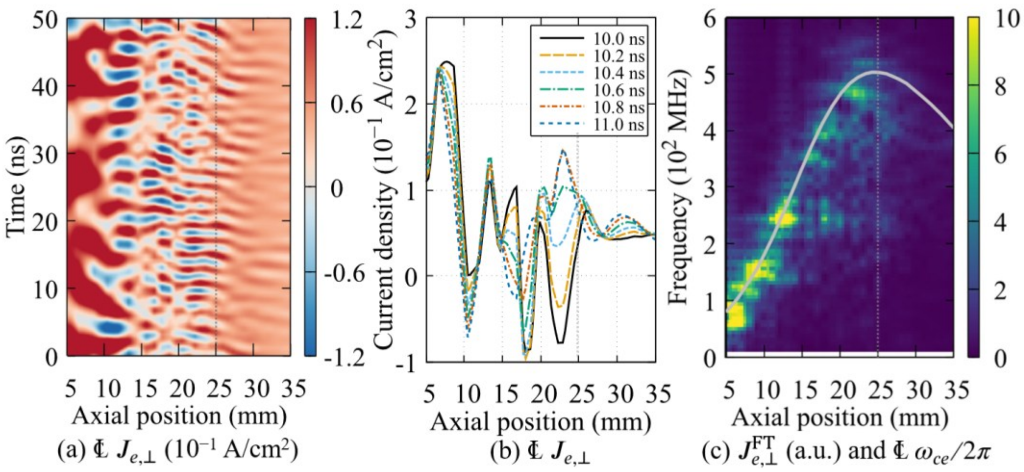

Figure 3. Temporal variation of $\boldsymbol{J}_{e,\perp}$ on the channel centerline (a), evolution of $\boldsymbol{J}_{e,\perp}$ on the channel centerline at 0.2 ns intervals (b), and $\boldsymbol{J}_{e,\perp}^{\mathbf{FT}}$ and $\boldsymbol{\omega_{ce}/2\pi}$ (grey line) (c)

Unlike conventional HET simulations, the fully coupled pressure-based method does not employ the generalized Ohm's law but instead directly solves the electron current conservation equation. This approach proves advantageous in that it enables simulations incorporating the electron inertia effect. To account for this effect, the time step is set to 1 ps. Figure 3(a) shows the temporal evolution of the perpendicular electron current density along the channel centerline, where persistent oscillations in the current density are observed. As the location approaches the anode, the oscillation period increases, while it decreases toward the channel exit, as clearly seen in Fig. 3(b). To investigate the spectrum of this oscillation, the perpendicular electron current density in the frequency domain is shown in Fig. 3(c), where the gray solid line indicates the electron cyclotron frequency. This suggests that the observed oscillations originate from the electron inertia effect and can be interpreted as electron cyclotron oscillations.

These oscillations occur in the direction perpendicular to the magnetic field applied in the HET, indicating that electron inertia, through electron cyclotron oscillations, may influence electron transport. A rigorous analysis of this phenomenon would require investigation of the plasma wave dispersion relation. However, since the present hybrid model assumes charge neutrality, the generation of electrostatic waves due to charge imbalance is precluded. Therefore, a more rigorous treatment would require incorporating charge imbalance or applying a perturbation method, which is left for future work.

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