A canonical view on particle acceleration by electromagnetic pulses

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In the present work, we investigate the dynamics of electrons under the action of wave-packets of high-frequency electromagnetic carrier waves. When the group velocities of the packets are subluminal, electrons can be efficiently accelerated. We show that the whole process can be described by an accurate ponderomotive canonical formalism that includes relevant extensions of the original ponderomotive approach applied to carriers moving at the speed of light. Single-particle simulations validate our analytical approach and show that extended canonical methods provides better agreement with numerics than previous investigations. In particular, we obtain a precise relationship between the wave amplitude and group velocity for optimum acceleration of initially stationary targets.

1. The physical model

We study the interaction of a relativistic electron with a modulated and polarized electromagnetic pulse, whose dispersion relation is of the hyperbolic type $\omega^2 = \omega_0^2 + c^2k^2$ The electron has p as the longitudinal (x direction) momentum. The fast phase of the pulse is described by $\theta = x - v\phi\ t$ with v_ϕ and $v_g=1/v_\phi$, respectively, the phase and group velocities normalized by c, the speed of light in vacuum. The normalized potential amplitude is a_0 and σ is the width of the packet, with $k\sigma$<<1 for slowly modulation.

Circular polarization

The relativistic Hamiltonians for the circular case,

$$H_{circ} = \sqrt{1 + p^2 + a_0^2 exp[-2(x - v_\phi t)^2/\sigma^2]} \qquad (1)$$

is an exact expression already free of the fast phase θ, so we have an exactly integrable Hamiltonian (time can be canonically absorbed into coordinate x) that coincides with its ponderomotive averaged form [1].

This leads to the conservation of the quantity K=H-vg t. By evaluating its value before and after the interaction between the electron and the wave packet - analogous to a collision - two possible outcomes for the electron's final velocity emerge: either *vf=0* (passing regime), or vf= [(v_ϕ + vg)/2]^(-1)(accelerative), assuming the electron is initially at rest.

In the latter case, one needs the particle to be effectively scattered forward. The critical condition guaranteeing that the particle will be continuously pushed forwardly, not going through the entire wave packet, is that $dx/dt = vg$ when the particle reaches the envelope peak at x-vg*t = 0 where the acceleration also vanishes. This, combined with the assumption $v_0 = 0$, imposes a minimum amplitude for scattering, which is therefore tied just to the phase velocity:

$$A_{0,min} = \frac{1}{\sqrt{v_\phi^{\ 2} - 1}} \tag{2}$$

Linear polarization

The relativistic Hamiltonian in the linearly polarized case,

$$H_{lin} = \sqrt{1 + p^2 + a_0^{\ 2}(1 - cos(2\theta))exp[-\ 2(x\ -\ v_\phi t\ )^2/\sigma^2]} \tag{3}$$

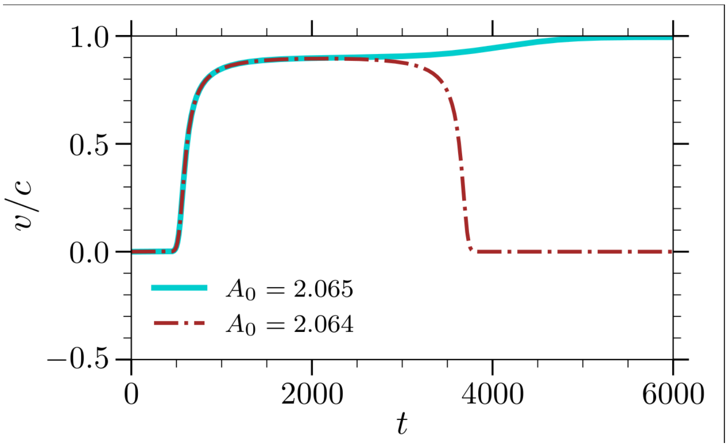
unlike in the circular case, is not naturally free of high-frequency terms. Thus, the constants of motion are only manifested by a ponderomotive canonical mean of the fast phase. The average Hamiltonian

$$\overline{H_{lin}} = \sqrt{\Gamma^2 - \frac{1-vg^2}{8}\frac{a_0^{\ 4}exp[-4(\bar{x} - vt\ )^2/\sigma^2]}{(\Gamma - vg\ \bar{p})^2}}, \tag{4}$$

onde $\Gamma^2 = 1 + p^2 + a_0^{\ 2}exp[-\ 2(\bar{x}\ -\ vt\ )^2/\sigma^2]$, describes the average dynamics of the particle in phase space better than the commonly used approximations [3], which do not have, for example, our additional fourth-order term in the potential amplitude.

The Hamiltonian (4) exhibits functionality similar to the circularly polarized case, and from the prominent conserved quantity it is possible to determine both the minimum potential amplitude that initiates the accelerated regime and the final electron velocity in this case.

Fig.1 Velocity versus time in circular case. Transition between both regimes passing (a_0 = 2.064, dash-dotted brown curve) and accelerating (a_0 = 2.064, solid cyan curve) for group velocity vg=0.9.



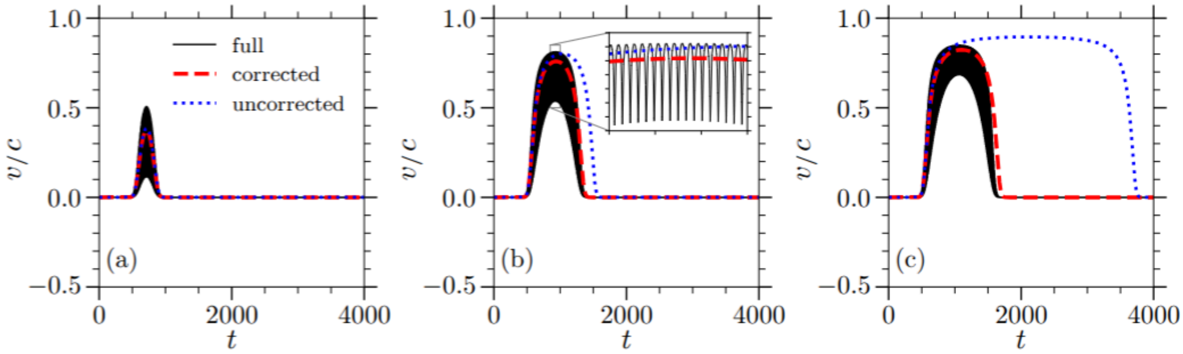


Fig. 2 Evolution of the agreement between analytics and numeric in the linear case. vg=0.9 and $\sigma = 1000$ in all cases, along with $a_0 = 1.0$ in panel (a), $a_0 = 1.9$ in panel (b), and $a_0 = 2.064$ in panel (c). Full simulations are depicted by the solid black line, the corrected model developed by authors by the dashed red line, and the original uncorrected model by the dotted blue line. Even at larger values of a_0 neighboring the transition from passing to reflected particles, the improved model keeps its nice agreement with simulations.

Ongoing Research

Currently, the possible effects of the **Radiation Reaction** (RR) in the accelerative scenario are being studied. The breaking of the conservation of ponderomotive invariants observed in the system that does not consider RR leads to different behaviors.

Studies indicate that, for wave packet frequencies in the ultra-X-ray region, **the minimum potential amplitude required for efficient acceleration can decrease** significantly, even if the final electron velocity is slightly lower than the initial value. This consideration is crucial for understanding acceleration by ultra-short pulses, where the interaction with the electron is so abrupt that radiation reaction (RR) effects cannot be neglected.

References

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