Hysteresis of amplitude-frequency relationship of MHD instabilities and its modeling in toroidal magnetically confined plasmas

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The relationship between the amplitude and frequency of magnetic fluctuations driven by MHD instabilities in toroidal magnetically confined plasmas has been investigated. It is known that, following a decrease in the frequency of magnetic fluctuations, a sudden increase in fluctuation amplitude can occur, leading to confinement degradation. In tokamaks, the locked mode may even trigger a disruption. Modeling the amplitude–frequency relationship could enable the prediction of the mode amplitude just before the mode locking, even without prior data on locked modes, by providing initial amplitude–frequency conditions corresponding to stable operation in future devices.

The characteristic behavior of mode frequency increase and decrease has been observed in locked-mode discharges in LHD[1] and in neoclassical tearing mode (NTM) discharges with applied ECCD in JT-60U[2]. In both cases, the trajectory in the amplitude–frequency diagram remains unique during both phases. This behavior is well explained by the force balance model, which captures the balance between a decelerating electromagnetic force and an accelerating viscous force [3]. Although the overall framework applies to both devices, the dominant decelerating force differs. In the case of locked-mode discharges in LHD, the main braking force arises from the electromagnetic force induced by externally applied resonant magnetic perturbations (RMPs) due to the RMP coils, whereas in NTM discharges of the JT-60U, it is attributed to the eddy-current-driven magnetic perturbation due to the instability (F_{rw}).

In LHD, repeated growth and stabilization of m/n=1/1 magnetic island have been observed in a relatively high-density regime [4]. m/n denotes the poloidal and toroidal mode numbers, respectively. For the first time, hysteresis behavior was discovered in the amplitude-frequency trajectory during the frequency deceleration and acceleration phases [5], as shown in Fig. 1. Red and blue symbols represent observation during the frequency decrease and increase phases, respectively. During the deceleration phase, the frequency gradually decreased as the amplitude increased. In contrast, in the acceleration phase, the frequency exhibits a sudden jump.

This study extends the force balance model to explain the hysteresis behavior observed in experiments, which cannot be captured by the conventional model. The model incorporates two key elements: diamagnetic drift and the frequency-dependent plasma response to the external RMP. In previous models, the mode frequency (ω_{mode}) was implicitly assumed equal to the E×B rotation frequency ($\omega_{\mathrm{E\times B}}$). Diamagnetic drift effect introduces a frequency offset between $\omega_{\mathrm{E\times B}}$ and ω_{mode}. Taking into account the frequency offset due to the diamagnetic drift frequency (ω_{dia}), the braking force becomes:

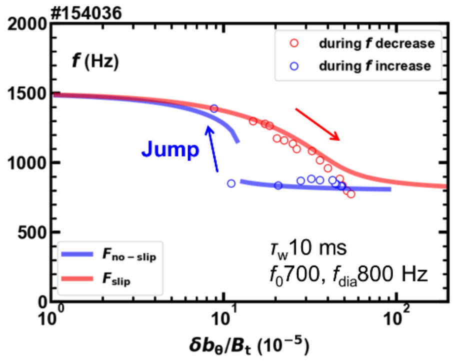

Figure 1. Hysteresis in the amplitude–frequency trajectory observed in LHD. Red and blue symbols indicate experimental data during frequency decrease and increase, respectively. The red and blue curves represent theoretical force balance solutions based on the braking force due to the plasma response field with slipping and non-slipping conditions, respectively.

$$F_{\mathrm{B}}(\omega_{\mathrm{mode}}) \equiv F_{\mathrm{B}}(\omega_{\mathrm{E\times B}} + \omega_{\mathrm{dia}}) \qquad (1)$$

As shown in the following equation, the driving force F_{D} depends on the E×B rotation frequency, not on the mode frequency.

$$F_{\mathrm{D}}(\omega_{\mathrm{E\times B}}) \equiv \mu(\omega_0 - \omega_{\mathrm{E\times B}}) \quad (2),$$

where ω_0 is plasma flow driven by some factor. Figure 2 illustrates the dependence of the decelerating and accelerating forces on the E×B rotation frequency, where the intersection point represents the quasi-steady-state frequency. Compared to the conventional model shown in Fig. 2(a), the model including the diamagnetic drift exhibits a shift in the braking force by ω_{dia}, as illustrated in Fig. 2(b).

Next, the plasma response field driven by the external RMP and its dependence on the plasma flow are considered. The plasma response field depends on the relative velocity between the external RMP and the plasma rotation. A large relative velocity corresponds to the slip regime, and is shown in Fig. 2(b) and is expressed by the following equation:

$$F_{\mathrm{B}} \equiv \delta b_{\mathrm{inst.}} \times \delta b_{\mathrm{RMP}} \frac{\omega\tau_{\mathrm{v}}}{\sqrt{1+\omega^2\tau_{\mathrm{v}}^2}} \qquad (3),$$

where $\delta b_{\mathrm{inst.}}$ is mode amplitude, δb_{RMP} is the plasma response field amplitude, τ_{v} is the wall constant time, and ω is the mode frequency. On the other hand, a small relative velocity leads to the no-slip regime. This condition is shown in Fig. 2(c) and is expressed by the

following equation:

$$F_B \equiv \delta b_{\text{inst.}} \times \delta b_{\text{RMP}} \frac{\omega\tau_v}{1+\omega^2\tau_v^2} \qquad (4)$$

In the no-slip model, it is noteworthy that under certain conditions, the solution exhibits

bifurcation, which can account for the observed sudden jump in the mode frequency.

The extended model, as shown in Fig. 1, reproduces the experimental observations well. The red and blue curves represent the theoretical force balance based on the plasma response field under slip and no-slip conditions, respectively. The same parameter values for f_0, f_{dia}, and τ_v are used for both deceleration and acceleration phases in the model. The wall time constant used in the model (τ_v ~ 10 ms) is a reasonable estimate for LHD. Since the external RMP is static, the relative velocity corresponds to the mode frequency. Therefore, the high-frequency deceleration phase is closer to the slip regime, while the acceleration phase is closer to the no-slip regime.

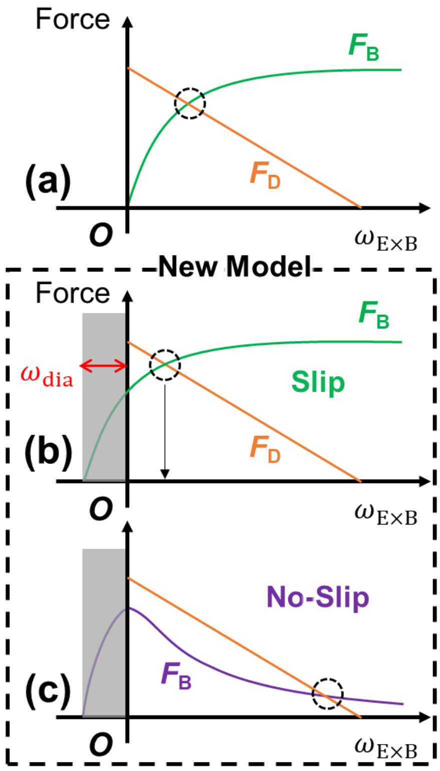

Figure 2. The dependence of the driving and braking forces on the E×B rotation frequency in (a) previous model, (b) slip model, and (c) no-slip model.

In JT-60U discharges where m/n=2/1 NTMs follow the onset of the resistive wall mode (RMW), a clear hysteresis in the amplitude–frequency trajectory has been newly identified, as shown in Fig. 3. Red and blue dots represent the observation during the frequency decrease and increase phases, respectively. While the frequency gradually increases during the acceleration phase, it drops abruptly during the deceleration phase.

The behavior during frequency decrease is well explained by the F_{rw} model, which corresponds to the no-slip model, and is expressed by the following equation:

$$F_B \equiv \delta b_{\text{inst.}} \times \delta b_{\text{inst.}} \frac{\omega\tau_v}{1+\omega^2\tau_v^2} \qquad (5)$$

The frequency at which the jump occurs is in good agreement with experimental observations. In contrast, the behavior during frequency increase cannot be explained by the no-slip model, as it predicts frequency jumps in both acceleration and deceleration phases within the low-frequency regime. As shown in Figure 3, the slipping model agrees well with the experimental data during the acceleration phase, using the same model parameters as the no-slip model for deceleration. The source of the plasma response field is not yet fully understood; possible candidates include non-resonant mode components and a weakly stabilized RWM.

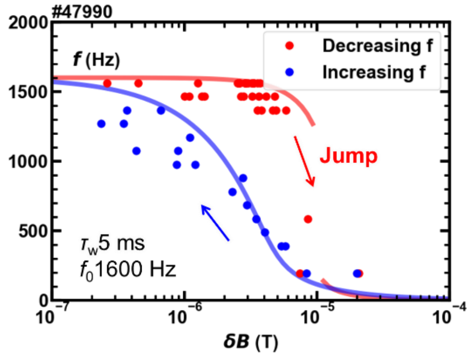

Figure 3. Hysteresis in the amplitude–frequency trajectory observed in JT-60U during NTM discharges with RWM. Red and blue symbols indicate experimental data during frequency decrease and increase, respectively. The red and blue curves represent theoretical force balance solutions under non-slipping and slipping conditions, respectively.

This study investigates the relationship between the amplitude and frequency of magnetic fluctuations associated with MHD instabilities under various conditions, including different driving sources of MHD instabilities and devices. Hysteresis in the amplitude–frequency trajectory was discovered during frequency acceleration and deceleration phases in both the LHD and the JT-60U tokamak. To explain this phenomenon, a new force balance model was developed, incorporating diamagnetic drift and the frequency-dependent plasma response to the external RMP. Remarkably, the developed model can reproduce the observed frequency behavior including hysteresis. In LHD, hysteresis is explained by the same type of braking force due to the external RMP coils with different slip conditions. In contrast, in JT-60U, hysteresis arises from a transition between different braking forces.

References

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