Cascaded parametric amplification of lower hybrid waves using microwave beams

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Introduction

Due to limited space usage, high efficiency and great precision, many planned fusion demonstration powerplants will rely on microwaves in the electron cyclotron range of frequencies (ECRF) for heating and current drive [1-3]. In such devices, the microwaves, generated by high-power gyrotrons, transfer energy to the electrons in the plasma at a harmonic of the electron cyclotron (EC) resonance. Logarithmically between the electron cyclotron range of frequencies and ion cyclotron range of frequencies lies the lower hybrid range of frequencies (LHRF). Waves in this frequency range can be used for efficient current drive.

We present a novel method for channelling energy to the LHRF in EC based devices by using two high-power gyrotron beams. The method, termed cascaded parametric amplification (CPA), is inspired by recent advances in nonlinear optics, where a similar technique is proposed to tackle the terahertz gap [4]. The idea of CPA is to launch two microwave beams such that they reach the upper hybrid (UH) layer with X-mode polarization, where they are converted to backward propagating electron Bernstein waves. We demonstrate that by overlapping the two gyrotron beams at the UH layer and spectrally separating them by approximately the lower hybrid (LH) frequency, a long cascade of parametric three-wave mixing processes can be initiated, resulting in a large energy transfer to a LH wave. The mechanism is analysed in one spatial dimension by solving a steady state coupled mode system. All involved plasma waves as well as the coupling coefficients are treated with kinetic theory. A detailed derivation of the system as well as benchmarking against kinetic particle-in-cell simulations will shortly be submitted for journal publication.

In this work, we describe the CPA mechanism and show an example where 10% of the gyrotron power is converted to a LH wave, thus beating the Manley Rowe limit. This suggests

that the CPA method could find relevance in magnetic confinement fusion experiments, e.g., during tokamak start-up.

Mechanism

At low beam power, the extraordinary mode (X-mode) converts linearly to a backward propagating electron Bernstein wave (EBW) at the UH layer. Such a conversion can be achieved by either launching an X-mode from the high field side (HFS) [5] or by using the double-conversion O-X-B scheme [6]. With high beam powers, e.g. using modern gyrotrons, power can be channelled from the heating beam to a downshifted EBW and a LH wave at the UH layer due to a parametric instability (PI).

PIs at the UH layer has been observed at several devices, both during direct HFS X-B conversion [7] and O-X-B heating [8,9]. PIs, which are a class of convective nonlinear three-wave interactions, become resonant when three linear modes in the plasma satisfy the phase matching conditions

$$\omega_n \approx \omega_{n-1} + \omega_L \qquad\qquad \boldsymbol{k}_n \approx \boldsymbol{k}_{n-1} + \boldsymbol{k}_L,$$

with, n, $n-1$, and L indexing the three interacting waves. An example of two EBWs and a LH wave satisfying the phase matching conditions is shown in the figure below.
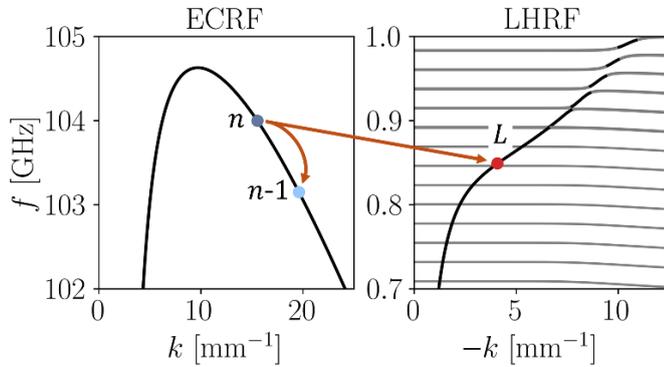
Figure 1: *1D example of three linear modes, marked by coloured dots, satisfying the phase matching conditions for a resonant PI. Here, a 104 GHz EBW, n, interacts with a 103.15 GHz EBW, n − 1, and a 0.85 GHz LH wave, L.*

The energy flow in a three-wave interaction is governed by the Manley-Rowe relations [10], which give rise to the three conserved quantities

$$M_n = \frac{I_{n-1}}{\omega_{n-1}} - \frac{I_L}{\omega_L}, \qquad M_{n-1} = \frac{I_{n-1}}{\omega_{n-1}} + \frac{I_L}{\omega_L}, \qquad M_L = \frac{I_L}{\omega_L} + \frac{I_n}{\omega_n}.$$

Thus, for the example above, a maximum of $\omega_L/\omega_n = 0.82\%$ of the power in the injected mode, n, can be channelled to the LH wave, L, while the remaining power goes to the downshifted EBW, $n-1$, in the case of a complete depletion of the injected wave.

The three-wave interaction described above is of the Stokes type, where a downshifted

EBW is generated. In general, anti-Stokes interactions will occur as well, where energy is channelled form a LH wave and an EBW to generate an upshifted EBW. The idea of CPA is to inject not only one, but two gyrotron beams spectrally separated with approximately the LH frequency. By overlapping the two beams at the UH layer and picking the gyrotron frequencies carefully, it is possible to stimulate a cascade of three-wave interactions. If the parameters are picked such that Stokes interactions are favoured over anti-Stokes interactions due to the phase matching conditions, substantial energy is channelled to the LH wave, vastly exceeding the Manley Rowe limit. It is important to note that the remainder of the energy is not lost but channelled to EBWs with frequencies slightly below the injected waves and will be damped at the Doppler shifted EC resonance, heating the electrons and potentially driving current.

The CPA mechanism can be modelled using WKB methods to describe the interacting waves as a coupled-mode system. A derivation of the system will not be covered here but will shortly be submitted for journal publication. The workflow is as follows: 1) The trajectories and wavenumbers of all interacting waves are computed in a 1D domain using ray tracing. 2) Coupling coefficients, group velocities, and linear collisional damping rates are computed with WKB methods. 3) The group velocities are adjusted to fit metaplectic geometrical optics [11] solutions to the wave envelopes, because some model assumptions break down at the UH layer. 4) Mode-conversions of the LH wave are computed by solving the tunnelling equation [12]. 5) A steady-state solution is found by solving a large coupled-mode system. This process can be iterated to find the optimal injected wave frequencies.

CPA example

An example of an output from a CPA simulation is shown below. In this scenario, around 10% of the total power in the system is channelled to the LH wave.
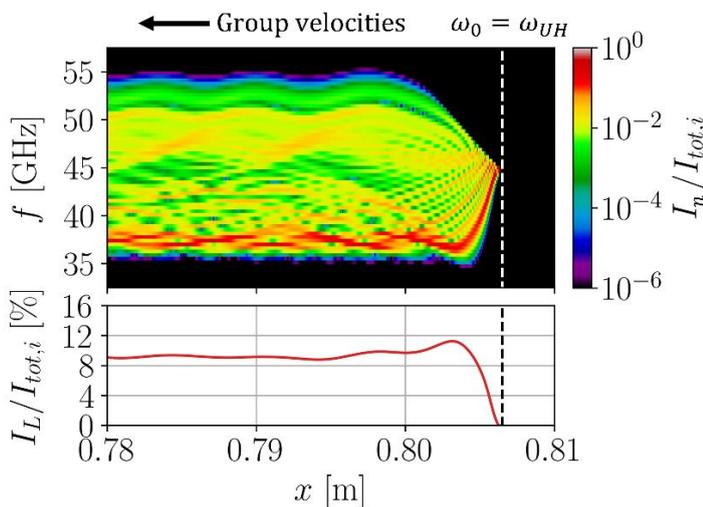
Figure 2: *Intensities in different waves in a CPA simulation. Two beams with frequencies $f_0 = 45\,GHz$ and $f_1 = 44.62\,GHz$, and intensities $I_0 = I_1 = 10\,kW/cm^2$ interact at the UH layer in a 1D domain resembling a typical pedestal profile in an ST40 H-mode plasma.*

Conclusion and outlook

We propose using the method of Cascaded parametric amplification to generate LH waves in magnetized plasmas by beating two gyrotrons at the upper hybrid layer. An example shows attainable power conversion efficiencies of 10%. With the remainder of the power going to EBWs, this mechanism could prove to be relevant for accessing the LH range in EC based fusion devices.

A detailed model derivation and benchmark will be submitted for journal publication shortly. Work remains in expanding the model to three dimensions and accounting for finite beam widths. The mechanism could be experimentally tested in small devices with multiple microwaves sources with tuneable frequencies. If successful, further tests could be carried out at devices with gyrotrons with first-harmonic heating capabilities by installing an additional low-power seed source, downshifted in frequency.

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