Crayon: a flexible fully relativistic ray tracer for electron Bernstein waves

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Electron Bernstein waves (EBWs) are an attractive option for microwave heating and current drive in spherical tokamaks (ST). STs operate at higher β_N and higher elongation κ compared to conventional aspect ratio devices and so could offer a pathway to compact, low magnetic field power plants with potentially reduced cost. Traditional microwave heating and current drive (HCD) using electron cyclotron (EC) waves is challenging in STs. The higher ratio of electron density n_e to magnetic field strength B means the fundamental cyclotron frequency is often inaccessible. Moreover, a large trapped particle fraction seriously degrades Fisch-Boozer current drive for low field side absorption. STs also have both steep ∇B and a well in B on the low field side due to a strong diamagnetic effect. Multiple cyclotron harmonics may appear and shield each other making achieving high field side absorption in the core challenging.

EBWs are quasi-electrostatic kinetic waves sustained by coherent electron gyration. They have no high density cutoff and show strong absorption even at low density and temperatures. EBWs damp on high energy electrons ($E \sim 10T_e$) resulting in up to a 3x enhancement in current drive efficiency compared to equivalent EC due to an efficient Ohkawa current driven from low field side absorption [1]. The HCD system is typically the largest consumer of recirculating power in power plant concepts. Consequently, maximizing the efficiency of the HCD system is key to delivering net electrical output.

EBWs do not exist in vacuum and must be coupled to within the plasma. The favoured method is O-X-B conversion. Here an O mode is launched at a critical angle such that the wave passes through its cutoff and is converted to a X mode. The X mode then propagates towards the upper hybrid layer where it naturally evolves into an EBW. The EBW then propagates into the over-dense ($f_{pe} > f$) plasma until it is absorbed due to cyclotron resonance. This process is illustrated in Figure 1.

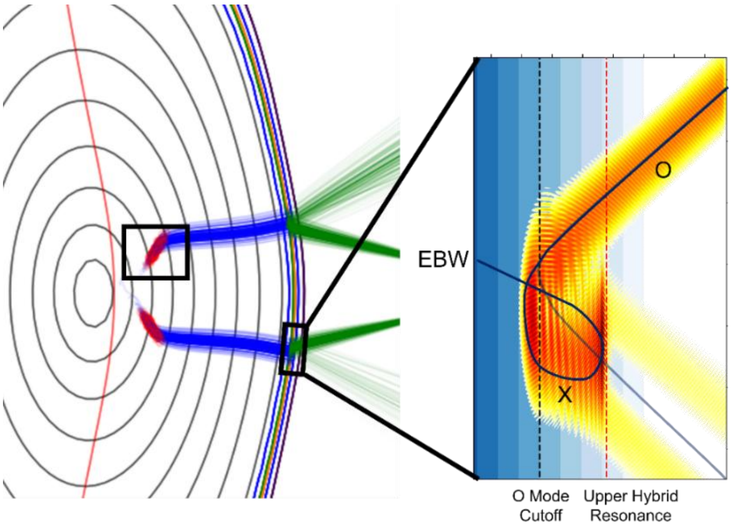

Figure 1: Left: Ray tracing simulation for MAST-U EBW system under installation. Right: Simulation of O-X conversion using the full wave code CUWA with illustrative paths of the incoming/reflected O mode, converted X mode and excited EBW. CUWA is a cold plasma code so the EBW wave field is not shown.

The EBW experimental physics base is much less mature than EC. To address this, MAST-U is installing 2 x 900kW, 5s 28/34.8 GHz gyrotrons aiming to operate in 2027. To support EBW physics on both MAST-U and STEP, a new ray tracing code called Crayon has been written to calculate EBW propagation, damping and current drive in fusion plasmas.

Ray tracing calculates the trajectories of the Poynting flux under the WKB approximation allowing calculation of the wave propagation and damping by solving a coupled set of ODEs of Hamiltonian type. EBW ray tracing has unique challenges compared to EC. For example, at power plant relevant temperatures, a fully relativistic plasm susceptibility is required to accurately predict wave trajectories. Very few existing codes implement kinetic propagation models and the only existing fully relativistic implementations tend to be very slow.

Computing the coupled power and the initial conditions of the converted X mode ray is crucial for accurate simulation of O-X-B conversion. However, O-X conversion occurs in regions where the WKB approximation is violated so it is unclear how it can be simulated in ray tracing. Existing implementations of O-X conversion use ad-hoc methods which lack a physics basis and are not robust for steep density gradients.

Other important physics include tunnelling between the X mode branches just after O-X conversion, collisional damping during the X-B conversion, higher order optical effects such as beam diffraction, etc. None of these are addressed for EBWs in any existing ray tracer hence a new code was necessary. Crayon features a fast fully relativistic dispersion relation solver for propagation and damping that is up to 100x faster than existing implementations

[2]. A linear current drive model accounting for momentum conserving collisions and finite Larmor radius effects allows rapid evaluation of current drive [3].

The O-X mode conversion is handled by asymptotically matching outgoing WKB trajectories with solutions of the local wave equation using a method originally developed for ICRH mode conversion [4]. This algorithm can detect and compute the mode conversion self-consistently on phase space only using the Hamiltonian and is illustrated in Figure 2. Mode conversion occurs when two eigenvalues of $\mathcal{H}$ become zero. Near this point the WKB approximation is not valid but becomes valid again along the outgoing rays.

The mode conversion is assumed to occur in an osculating plane where $\mathcal{H}$ has a saddle structure. The outgoing WKB branch is detected by finding roots of $\mathcal{H} = 0$ along the line from the incoming WKB branch through the saddle point. Mode conversion is detected when the ray passes a minimum in the 2nd invariant of the dispersion tensor. This is more robust for steep density gradients than using a density threshold (Figure 2).

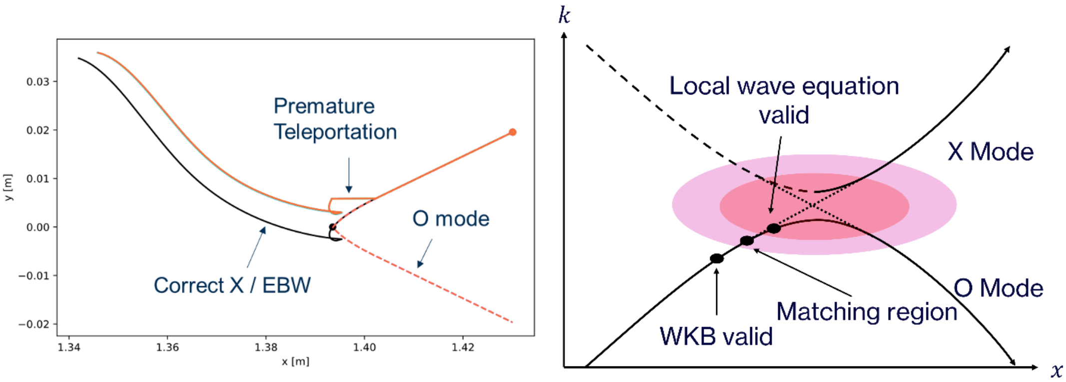

Figure 2: Left: ray tracing with O-X mode conversion (right) in GENRAY (orange) compared to Crayon (black). Right: illustration of the asymptotic matching at the O-X mode conversion.

Crayon can automatically switch between a cold, warm non-relativistic and fully relativistic model depending on the local ray parameters. For high temperature plasmas in STEP the fully relativistic model is required but for relatively colder plasmas like in MAST-U the nonrelativistic warm model can be used.

Calculating the ray trajectory requires derivatives of $\mathcal{H}$ in phase space. However, numerical derivatives of kinetic dispersion relations can be inaccurate in often encountered parameter regimes so analytic formulas are required. For example, the second derivative of the plasma dispersion function $Z''(\zeta) = -2(Z + \zeta Z') \to -2/\zeta^3$ for large ζ. However, due to finite floating-point precision the computed value will be inaccurate (Figure 3).

Previous attempts at analytic derivatives used computer algebra packages resulting in code which is hard to read or debug. However, the calculation can be made easier by applying the Jacobi formula, which relates the derivatives of a determinant to derivatives of the tensor elements, and the Faa di Bruno formulas, which allows calculation of the chain rule as array contractions. The analytic derivatives of the warm non-relativistic susceptibility can then calculated using a modular, readable and testable code base.

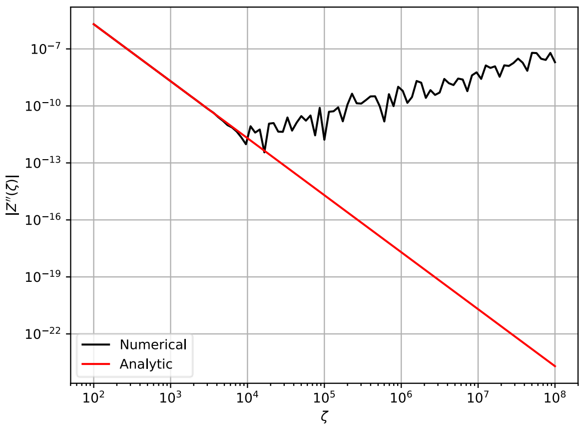

Figure 3: Real part of second derivative of plasma dispersion function $Z''(\zeta)$ computed using finite differences (black) and an analytical formula including large argument asymptotic expansions (red).

Ray tracing requires plasma parameters defined in distinct coordinate systems. For example, in tokamaks we may solve ray trajectories in Cartesian, define an axisymmetric magnetic field in cylindrical and n_e and T_e as 1d functions of a flux coordinate. This geometry is often hard coded into ray tracing codes which restricts the cases to which it can be applied.

In contrast, Crayon uses a generalised approach to coordinate systems where input parameters are constructed in their defined coordinate and then mapped to Cartesian. To map a generic tensor field from coordinate system $p \rightarrow q$ we contract with the forward transform $F_{\mu}^{\cdot\nu} = \partial p^{\nu}/\partial q^{\mu}$ for each covariant index and the backward transform $B_{\mu}^{\cdot\nu} = \partial q^{\nu}/\partial p^{\mu}$ for each contravariant index. To calculate spatial derivatives, the covariant derivative is constructed in the local coordinate system which can then be mapped to cartesian as above. Crayon allows EBWs to be studied in start-up where the density and temperature are defined in cylindrical coordinates, or stellarators where the magnetic field is no longer axisymmetric.

Acknowledgement

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