Study of the density limit physics for stellarator devices by means of an energy balance model

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Abstract

In stellarator devices, there is an upper limit for plasma density related to radiative collapse. In the present work, we develop a physics model to predict the density limit, relying on the global energy balance equation in stationary state. The model is well represented by a scaling law that, while device-specific, shows little variation across machines. We have tested the model against TJ-II experimental data, observing that it reproduces well the dependency with heating power. However, it systematically overestimates the measured density limit, likely due to approximating the radiation model by coronal equilibrium.

1. Introduction

When the density reaches a maximum value, n_{lim}, the plasma suddenly collapses. Even though the fusion community is aware of this phenomenon, the physics mechanisms causing this limit are not yet fully understood. For stellarators, the density limit is thought to be related to radiative collapse, as described in [2]. The most common approach for predicting the density limit is through empirical scalings, such as the Sudo scaling [4]:

$$n_S[10^{20}m^{-3}] = 0.25\sqrt{\frac{P_\mathrm{h}[MW]B[T]}{a[m]^2R[m]}} \qquad (1)$$

Where P_h is the heating power absorbed by the plasma, B is the magnetic field, and a, R are the minor and major radii. While empirical scalings are useful for quick estimates, they do not offer insights into the physical mechanisms behind the limit. In contrast, physics-based models permit one to gain a deeper understanding of physics.

This work focuses on the development of an energy balance model (EBM) based on [2] to predict the density limit in stellarators. The model assumes that the limit arises when heating power can no longer compensate for the combined radiation and transport losses, leading to thermal instability. First, the EBM is presented. Then, it is shown that the model results can be expressed as a scaling law. Finally, the model is tested against TJ-II experimental data.

2. Energy Balance Model

Consider an experiment where the density increases steadily at constant heating power. As the density rises, the plasma temperature drops, leading to increased radiation losses (due to enhanced line radiation at lower temperatures). The density limit is defined as the value at which radiation losses become so large that the heating power can no longer maintain balance, triggering a thermally unstable regime (the so-called radiative collapse).

The energy balance equation is written, in its global form, as:

$$\frac{P_{\mathrm{h}}}{V} = n_{\mathrm{e}}^2 \sum_z (c_z L_z) + \frac{3 n_{\mathrm{e}} T}{\tau_{\mathrm{E}}} \qquad (2)$$

Being V the plasma volume, n_{e} the electron density, $c_z = n_z / n_{\mathrm{e}}$ the concentration of impurity z, L_z the cooling factor of such impurity, T the plasma temperature (which is assumed to be equal for ions and electrons) and τ_{E} the energy confinement time. Equation (2) is the expression used in the EBM for establishing the power balance. By plotting in figure 1 each term of equation (2) with respect to the average temperature while maintaining n_{e} and c_{imp} constant, we identify two regions: a transport dominated, thermally stable branch, and a radiation dominated, thermally unstable branch. The minimum heating power required to sustain the plasma is found between the two branches. This minimum point is also the boundary between stable and unstable conditions, and therefore the starting point of the radiative collapse. Considering from equation (1) that the required heating power increases monotonously with density, the pair $(P_{\mathrm{min}}, n_{\mathrm{e}})$ corresponds to the density limit state.

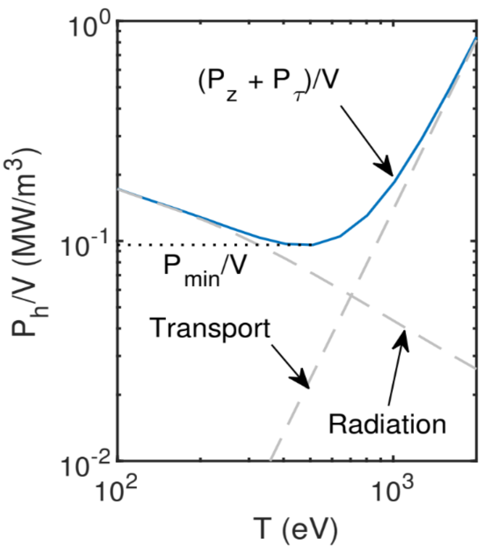

Fig. 1. Global power balance as a function of averaged temperature. The grey dashed lines are transport (P_τ) and radiation (P_z) losses. The blue solid line is the needed heating power.

The model incorporates several assumptions. The cooling factors used in equation (2) are calculated under coronal equilibrium approximation, which assumes local balance between ionization and recombination, ignoring impurity transport across temperature gradients [3]. The energy confinement time is computed following the ISS04 scaling law [6], commonly used for stellarators. While such assumptions are common in simplified physics models (as in [1], [5]) they introduce limitations. The coronal equilibrium is typically less accurate in the plasma edge, where the impurity transport time is smaller than ionization and recombination times. The ISS04 scaling law was derived from a database of low-radiation plasmas, and therefore it is less accurate for highly radiative plasmas. The model must be therefore verified by comparing it with experimental scalings and data, as will be shown in the next sections.

3. Scaling laws

The most typical form of expressing the density limit is by means of a scaling law [4]. Therefore, it is interesting to check if the n_{lim} results yielded by the EBM can be expressed in the form of a scaling law, to compare it with the Sudo limit. We have chosen the parameters from equation (2) that most typically vary in plasma discharges, namely the heating power, magnetic field and impurity concentration. Therefore, the proposed scaling law is:

$$n_{\mathrm{lim}} = n_0 \left(\frac{P_{\mathrm{h}}}{V}\right)^{\alpha} B^{\beta} c_{\mathrm{imp}}^{\gamma} \qquad (3)$$

	***n*0**	***α***	***β***	***γ***
CTH	1.12·10^20	0.58	0.41	-0.37
TJ-II	1.39·10^20	0.56	0.31	-0.41
HSX	1.53·10^20	0.56	0.30	-0.42
W7-X	1.95·10^20	0.57	0.38	-0.41
LHD	2.37·10^20	0.58	0.41	-0.39

Table 1. Scaling law exponents for different devices.

By solving the model, a database of the density limit can be built for different conditions in P_{h}, B, and c_{imp}, and for given device parameters R, a, $\iota_{2/3}$. For the sake of simplicity, the impurity mix and the density and temperature radial profiles were kept constant for all devices. Namely, the impurity mix was chosen as 49% O, 49% C, 0.5% Fe and 0.5% Cr. The resulting database can be then used for fitting the exponents of the scaling law. The obtained coefficient of determination was, in all cases, $R^2 > 0.9$, indicating that equation (3) is indeed a good representation of the model. Table 1 shows the exponents obtained for five different devices: CTH, TJ-II, HSX, W7-X and LHD. The heating power exponent α closely aligns with that in the Sudo scaling. The magnetic field exponent β is also not very different to the dependence of the Sudo limit with B, but it shows more variability. The baseline density n_0 increases with machine size, indicating a direct influence of device dimensions on the density limit. This fact can be easily explained by the increase of τ_{E} with machine size, and therefore the consequent reduction in transport losses.

4. Experimental validation with TJ-II data

The EBM was tested using 72 discharges extracted from the TJ-II database. All selected discharges finished due to reaching the density limit. The discharge parameters needed as inputs for the model are $B = 1$ T for all the discharges, $P_{\rm h} \in (200, 630)$ kW/m^3, and $c_{\rm imp} = 7\%$. In reality, $c_{\rm imp}$ varies for each discharge, but we lack the diagnostics to measure it precisely. Therefore, we had to choose a constant value consistent with the typical observed $Z_{\rm eff} \sim 2$. The impurity mix and radial profiles were also kept the same as those used in deriving the scaling laws.

Figure 2 compare the density limit predictions with the experimental measurements. The model follows the experimental tendency of the density limit. However, when analyzing each NBI cluster separately, the tendency disappears. This disagreement may arise due to variations in $c_{\rm imp}$ that are not reflected in the data. It can be seen as well that the model systematically overestimates by 20% the density limit, which may be caused by the corona approximation, since coronal equilibrium tends to underestimate the radiative losses, which may in turn overestimate the density limit.

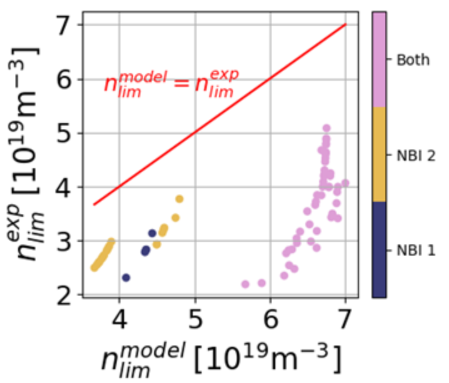

Fig. 2. Comparison between density limit EBM predictions and experimental measurements. Blue data corresponds to discharges heated by NBI 1, yellow data to NBI 2, and pink data to both NBIs.

5. Conclusions

We have developed an energy balance model for estimating the density limit in stellarators that is also well represented by a scaling law, and its dependency with heating power is close to the Sudo limit. Furthermore, the exponents in the scaling law are approximately device-independent. When testing with TJ-II data, the model reproduces experimental database tendencies, although it also tends to overestimate $n_{\rm lim}$. Future work will include implementing more realistic cooling factors (non-corona) and testing with different devices.

References

[1] J.A. Alonso et al., *Nucl. Fusion* 62, 036024 (2022).
[2] K. Itoh et al., *J. Phys. Soc. Jpn.* 70, 3274 (2001).
[3] D. Mosher, *Phys. Rev. A* 10, 2330 (1974).
[4] S. Sudo et al., *Nucl. Fusion* 30, 11 (1990).
[5] H. Vernickel et al., *Nucl. Fusion* 18, 1467 (1978).
[6] H. Yamada et al., *Nucl. Fusion* 45, 1684 (2005).