Particle Transport and Turbulent Cascade Rate in the Scrape-Off Layer

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Abstract

The comprehension of turbulent transport at the edge of fusion plasma devices is of crucial importance for the optimization of magnetic confinement. In this work, we studied the dynamics of blob-like structures in the region of the scrape-off layer using a two-dimensional Braginskii model and, following the Yaglom-Monin approach, we derived a new third-order exact law that describes the turbulent cascade in fusion plasmas.

Introduction

Turbulent phenomena in the Scrape-Off Layer (SOL) of fusion devices play a critical role in determining magnetic confinement. This region is characterized by the presence of high-density and high-temperature structures, known as filaments or "blobs", which propagate outward toward the device wall, thereby degrading confinement [1]. In this work, we investigate the turbulent plasma dynamics in the SOL using a two-dimensional Braginskii model, which describes the formation and evolution of blob structures. We study these complex dynamics using both classical Eulerian analysis and a Lagrangian approach, by following fluid particles injected at the shear layer. Moreover, we extend the analysis beyond classical second-order statistics, proposing a novel high-order turbulence law that directly quantifies the turbulent cascade rate of density fluctuations in the edge-SOL region, for the electrostatic Braginskii model.

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The Turbulence Model and the Yaglom-Braginskii relation

The turbulence model is based on the electrostatic drift-reduced Braginskii equations [2], which evaluates the time evolution of the plasma density n, electron temperature T, and vorticity Ω, in a 2D geometry, under the assumption of quasi-neutrality, cold ions, and isotropic pressure:

$$\frac{dn}{dt} + nC(\phi) - C(nT) = \nu_n \nabla^2 n - \sigma_n(x)(n-1), \tag{1}$$

$$\frac{dT}{dt} - \frac{7T}{3}C(T) - \frac{2T^2}{3n}C(n) + \frac{2T}{3}C(\phi) = \nu_T \nabla^2 T - \sigma_T(x)(T-1), \tag{2}$$

$$\frac{d\Omega}{dt} - C(nT) = \nu_\Omega \nabla^2 \Omega - \sigma_\Omega(x)\Omega, \tag{3}$$

$$\Omega = \nabla^2 \phi, \tag{4}$$

The total time derivative is defined as $\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{u}_E \cdot \nabla$, where the advection is due to the electric drift velocity $\mathbf{u}_E = \frac{\mathbf{E}\times\mathbf{B}}{B^2}$, and the curvature operator $C(\bullet)$ accounts for the radial magnetic field gradient. Applying the Yaglom-Monin procedure [3] of spatial increments to the continuity equation (1), in a decaying turbulence regime, we can derive an exact third-order turbulence law for the particle density, providing a scale-by-scale budget for the cascade of density fluctuations. The new Yaglom-Braginskii third-order law reads

$$\varepsilon = -\frac{1}{4}\frac{\partial}{\partial t}\left\langle (\delta n)^2 \right\rangle + \frac{1}{2}\left\langle \bar{\chi}(\delta n)^2 \right\rangle - \frac{1}{2}\zeta \left\langle \delta n \delta(nE_y) \right\rangle - \frac{1}{2}\zeta \left\langle \delta n \delta G_y \right\rangle$$
$$-\frac{1}{4}\frac{\partial}{\partial r_j}\left\langle \delta u_j (\delta n)^2 \right\rangle + \frac{1}{2}\frac{\partial^2}{\partial r_k^2}\langle \nu(\delta n)^2 \rangle, \tag{5}$$

where $\varepsilon = \langle \nu(\partial_k n)^2 \rangle$ is the turbulent dissipation rate of the density with ν the dissipation coefficient. The increment $\delta n = n(\mathbf{x}+\mathbf{r}) - n(\mathbf{x})$, is defined over a spatial lag r, and $\bar{\chi}$ quantifies the system's compressibility. Each term of Eq. (5) represents a distinct physical effect. The first term on the right-hand side captures the temporal unsteadiness of the cascade at large scales, which provides energy to the system, as well as the second, the third, and the fourth terms which quantify large-scale inhomogeneity due to velocity compressibility, magnetic field curvature, and pressure balance shear. The most relevant contribution is the Yaglom term, $\frac{\partial}{\partial r_j}\left\langle \delta u_j (\delta n)^2 \right\rangle$, which represents the nonlinear transfer of density fluctuations across scales, defining the inertial range of turbulence. Dissipation at small scales is captured by the last term. More details on the turbulence model and Yaglom-Braginskii relation can be found in [4].

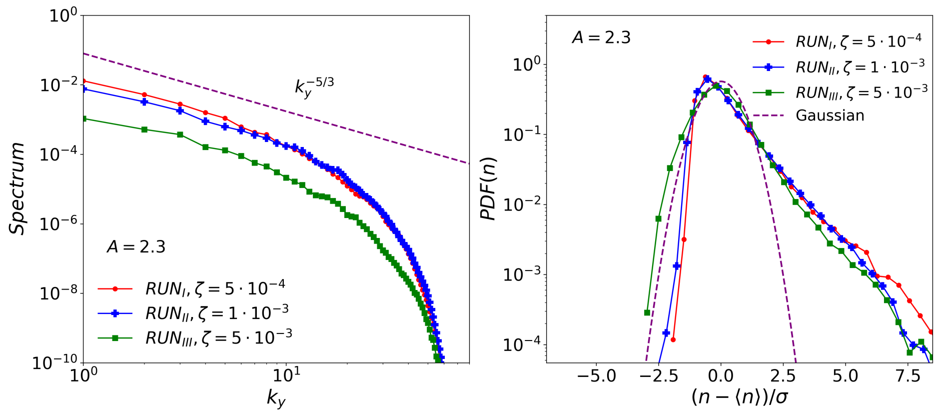

Figure 1: Comparison between the average energy spectrum and the PDF of particle density for different magnetic shear values. The results indicate that the system is more stable and less intermittent for higher magnetic shear (larger ζ).

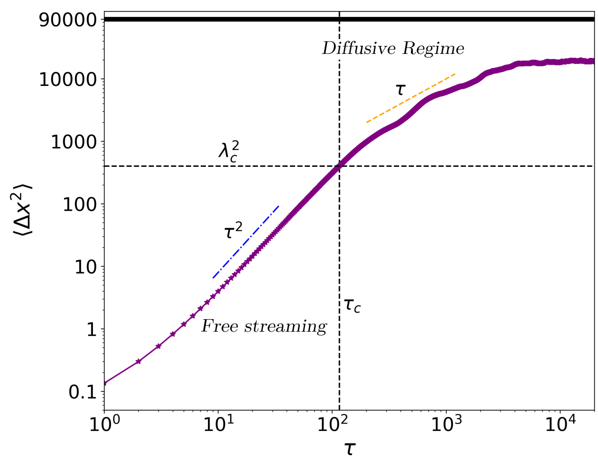

Figure 2: Temporal evolution of the MSD of fluid elements, in a double logarithmic scale. The dashed lines represent the square of the correlation length λ_c and the correlation time τ_c. The solid black line marks the maximum allowed displacement.

Results and Conclusions

To investigate different turbulent regimes we used Eqs. (1-4) to perform a campaign of simulations varying the radial gradient of the magnetic field. Following an Eulerian approach, in figure 1, we compare the averaged spectra and the probability distribution function (PDF) of the plasma density n, for each simulation, highlighting that higher magnetic shear leads to more stable and less intermittent configurations. In the second part of our analysis, following a Lagrangian approach, we investigated the diffusion of fluid elements in the SOL by tracking their trajectories in the numerical domain. To characterize the transport dynamics, we evaluated the mean square displacement (MSD) of the radial positions $\langle \Delta x^2 \rangle$, shown in figure 2 as a function of time. In the initial stage of their journey, fluid tracers experience a free-streaming motion,

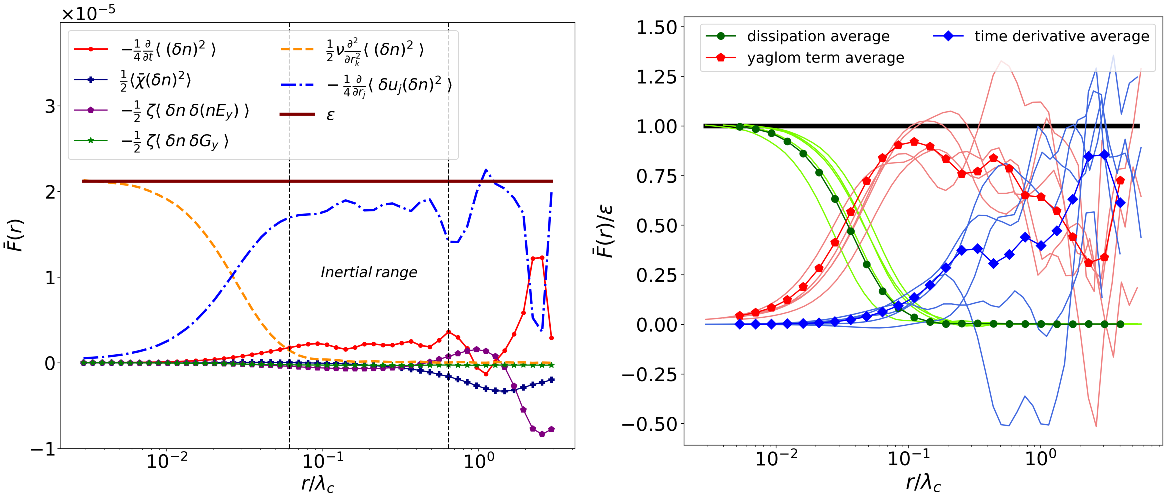

Figure 3: (left panel) Polar average of each term of the Yaglom-Braginskii relation Eq.(5) as a function of the normalized scale; (right panel) Comparison of the main terms of the third-order law at different times. Averages are reported with lines and symbols.

with the MSD scaling as τ^2, followed by a diffusive behavior with $\langle \Delta x^2 \rangle \propto \tau$. This transition occurs when the displacement becomes comparable to the turbulence correlation length λ_c, namely the size of the blobs. Finally, we performed new numerical simulations, in a decaying turbulence regime (without any forcing term), to validate the new Yaglom-Braginskii law. The turbulence cascade law in Eq.(5) is shown in figures 3, where a clear separation of scales is evident and few main contributions dominate at distinct regimes. At large scales, $r > \lambda_c$, the dynamics are dominated by injection mechanisms, including instabilities driven by large-scale gradients and contributions from magnetic field inhomogeneities and the electrostatic field. The intermediate scales, corresponding to a well-defined turbulent inertial range, exhibit a Yaglom-like energy transfer term, Finally, at small scales, the cascade is terminated by the viscous (dissipative) terms.

References

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